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Laser applications to the study of atomic quantum structure.

2017 OSA Siegman International School on Lasers,
CIO, León, México, August 2017

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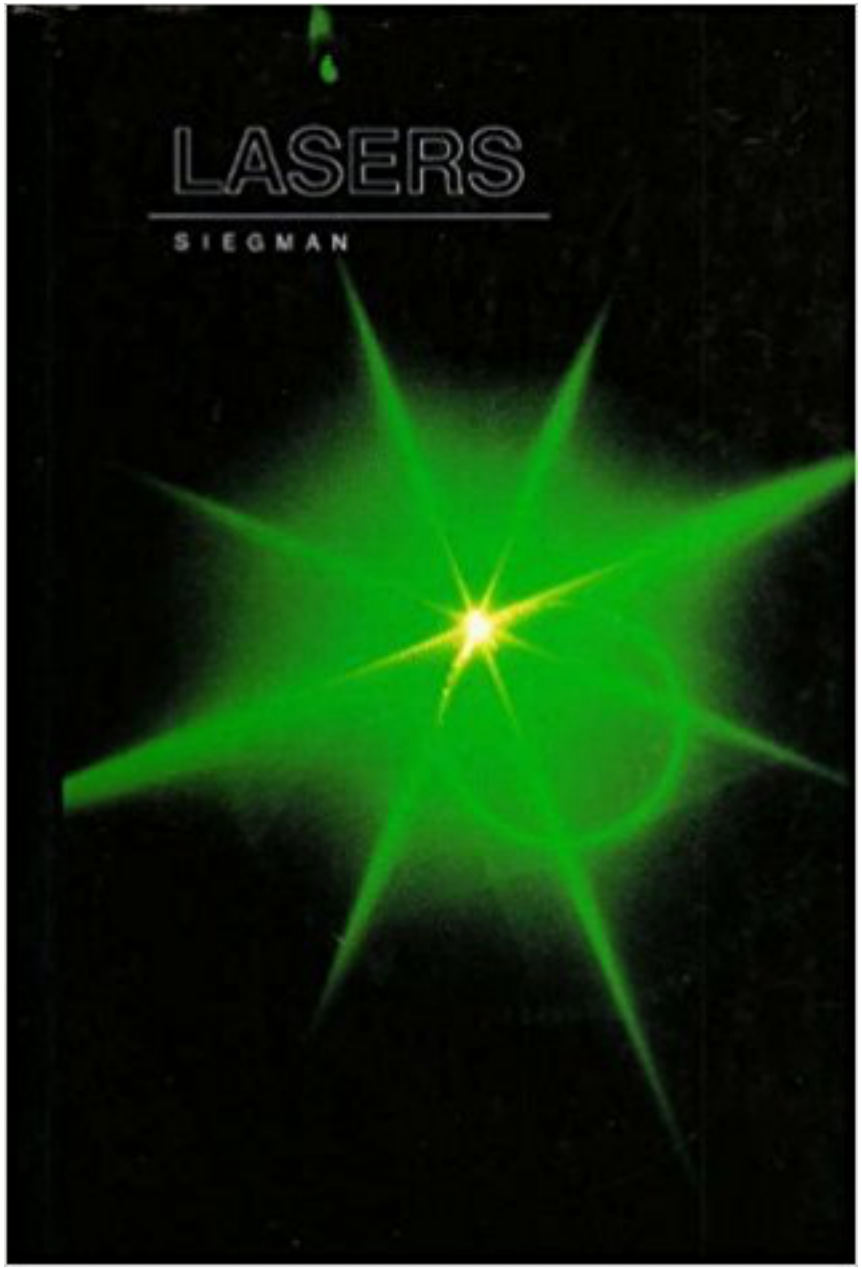
PHYSICS FRONTIER CENTER

NIST



Anthony E Siegman
(1931-2011)

Plan of the course:

1st lecture: Introduction to the interaction of light with atoms, (nanofibers).

2nd lecture: Atom-light interaction of a two level atom, (nanofibers and cavity QED).

3rd lecture: Different types of laser traps for atoms, (nanofibers, cavity QED, and spectroscopy)

4th lecture: Real atomic structure in Rb and Fr.

5th lecture: Weak interaction studies with Fr, a proposal.

1st lecture: Introduction to the interaction of light with atoms, (nanofiber examples).

Bibliography 1st lesson. Review Article:

P. Solano, J. A. Grover, J. E. Hoffman, S. Ravets, F. K. Fatemi, L. A. Orozco, and S. L. Rolston “Optical Nanofibers: A New Platform for Quantum Optics”. Advances in Atomic Molecular and Optical Physics, Vol. 46, 355-403, Edited by E. Arimondo, C. C. Lin, and S. F. Yelin, Academic Press, Burlington (2017).

ArXiv:1703.10533

1. A review of Electricity and Magnetism

Maxell's Equations:

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial (\varepsilon_0 \mathbf{E} + \mathbf{P})}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0.$$

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$$\nabla \cdot \mathbf{B} = 0.$$

Wave Equation:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

$$-\nabla(\nabla \cdot \mathbf{E}) + \nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}.$$

In free space, $\nabla \cdot \mathbf{E} = 0$ and $\mathbf{P} = 0$,

$$\nabla^2 \mathbf{E} - \frac{1}{v_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

A note about polarization

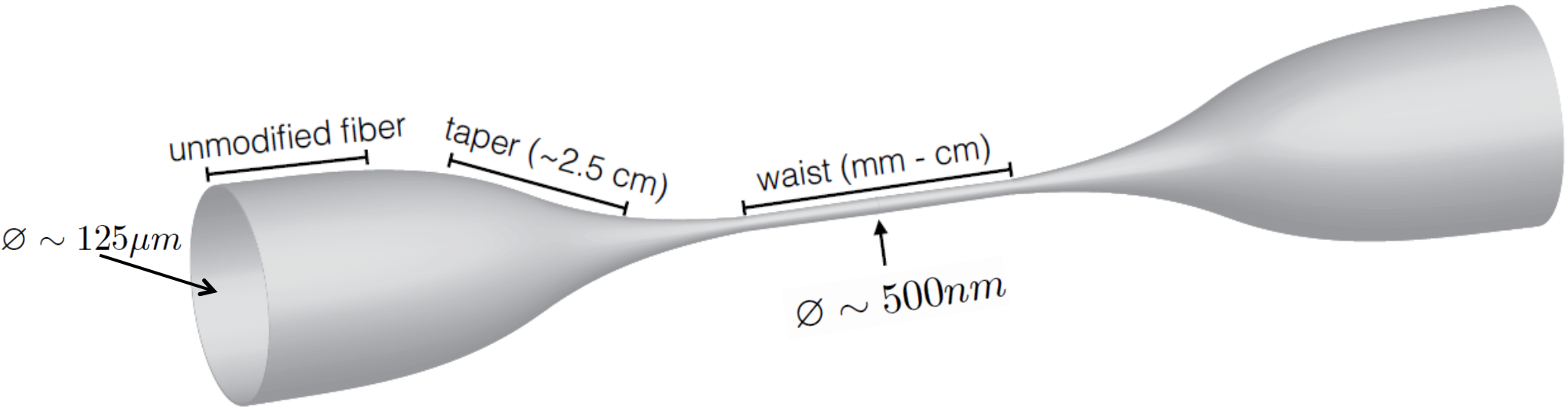
Gauss's Law in free space:

$$\nabla \cdot \vec{E} = 0$$

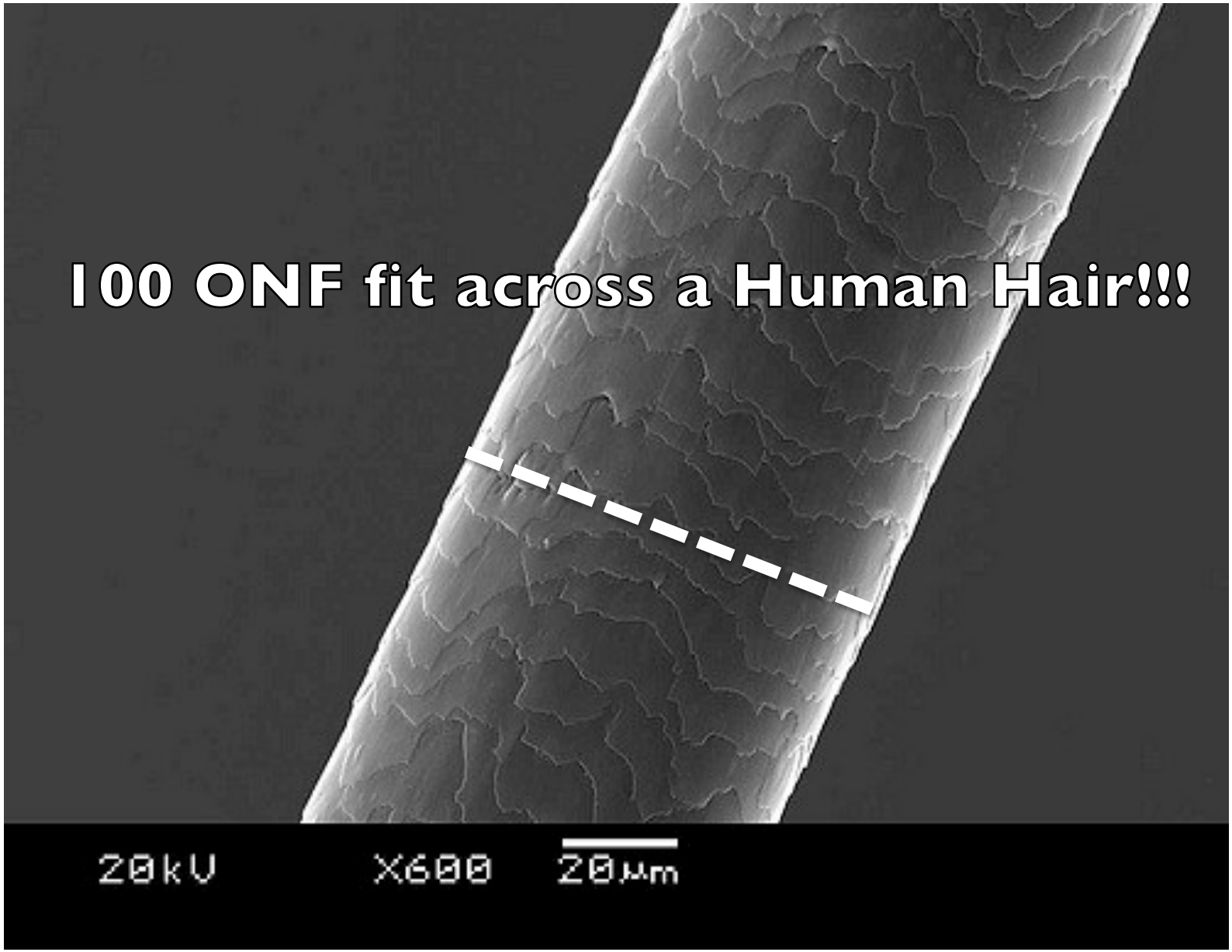
$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

If there is a "transverse gradient" in the radiation field propagating in *z*, there is a longitudinal polarization also in *z*

Optical Nanofibers

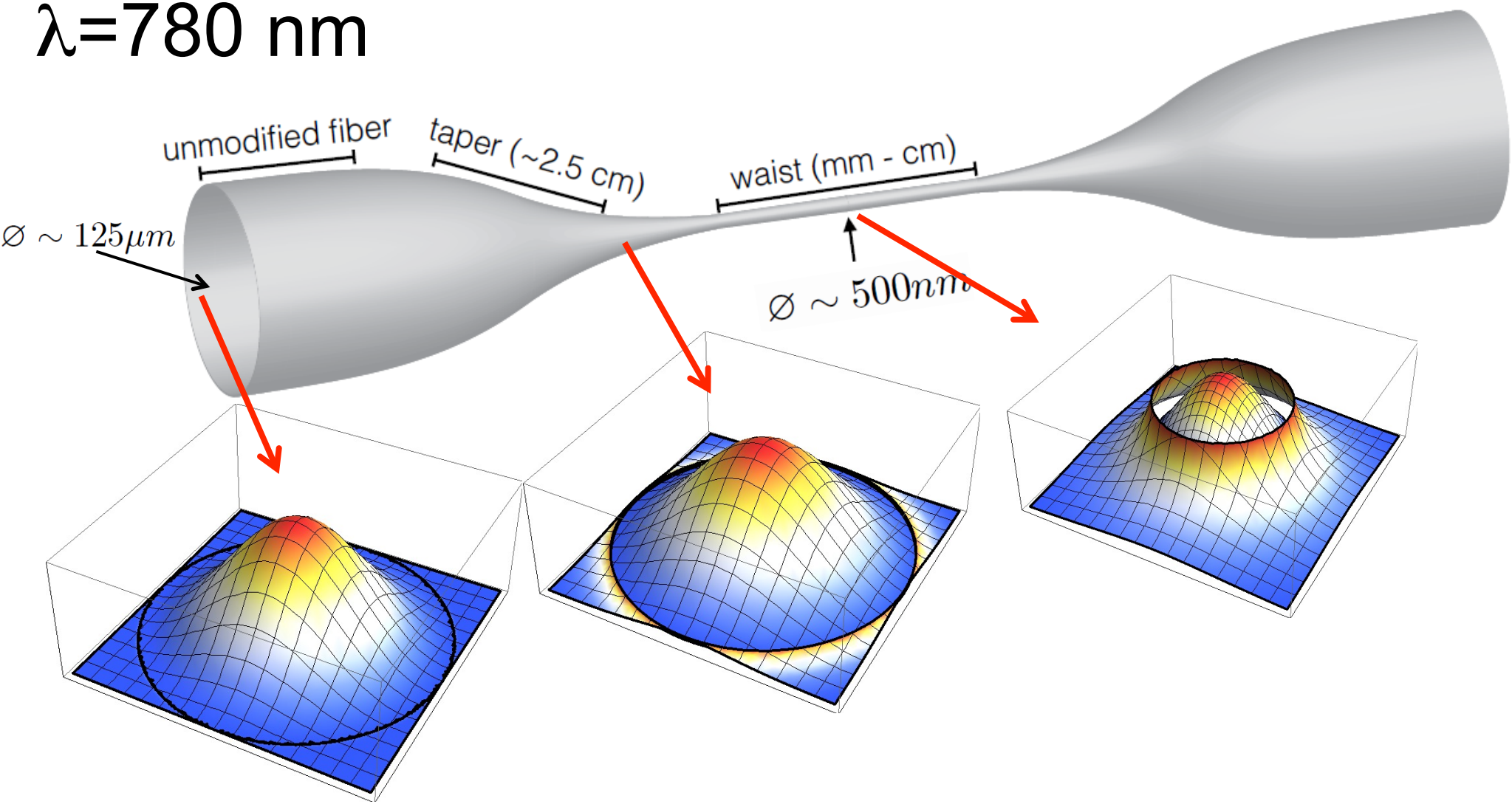


Making Sense to the Scale



Optical Nanofibers

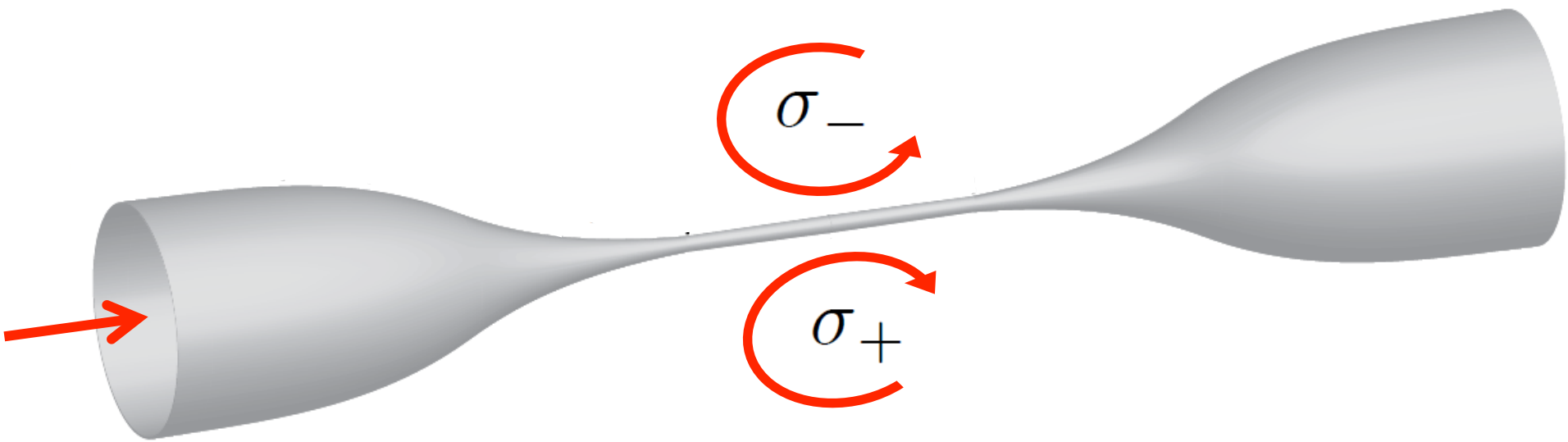
λ=780 nm



Polarization at the fiber waist

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$



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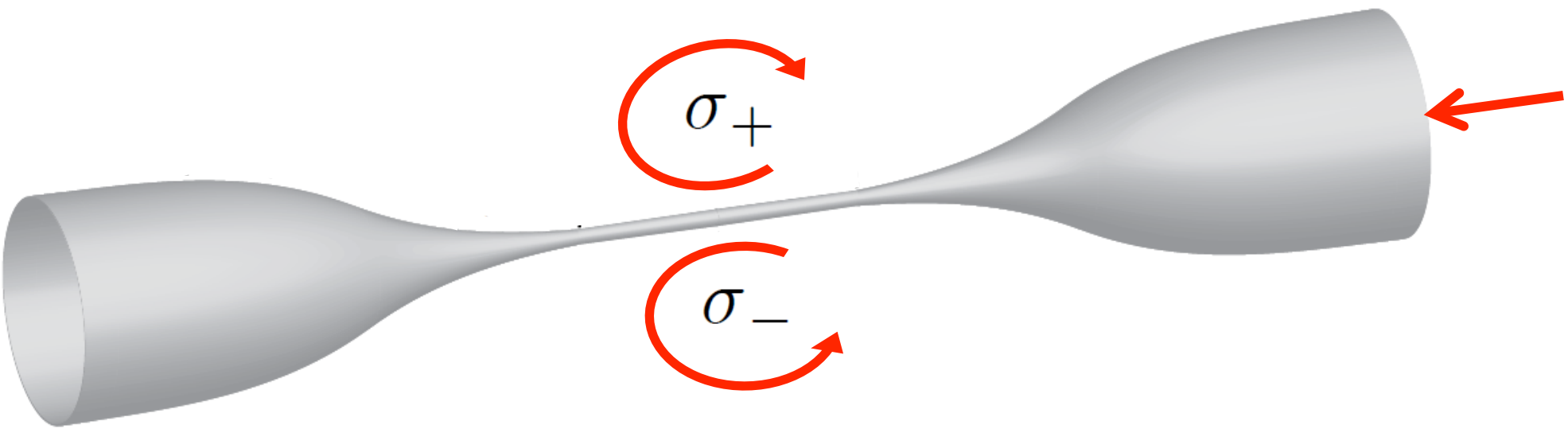


2. Lorentz model of the atom

The atom of Lorentz: the electron oscillating around the heavy ion

$$m\ddot{\mathbf{x}} + m\omega_0^2\mathbf{x} = 0$$

Driven by a monochromatic field:

$$\mathbf{E}^{(+)}(t) = \hat{\varepsilon}E_0^{(+)}e^{-i\omega t}$$

Driven harmonic oscillator

$$m\ddot{\mathbf{x}}^{(+)} + m\omega_0^2\mathbf{x}^{(+)} = -\hat{\varepsilon}eE_0^{(+)}e^{-i\omega t}$$

$$\mathbf{x}^{(+)}(t) = \hat{\varepsilon} x_0^{(+)} e^{-i\omega t}$$

$$-m\omega^2 x_0^{(+)} + m\omega_0^2 x_0^{(+)} = -eE_0^{(+)}$$

There is a resonance (divergence) and a change of phase

$$x_0^{(+)} = \frac{eE_0^{(+)}/m}{\omega^2 - \omega_0^2}.$$

The atomic dipole then is:

$$\mathbf{d}^{(+)} = -e\mathbf{x}^{(+)}$$

With damping (the charge is accelerated so it radiates):

$$m\ddot{\mathbf{x}}^{(+)} + m\gamma\dot{\mathbf{x}}^{(+)} + m\omega_0^2\mathbf{x}^{(+)} = -\hat{\varepsilon}eE_0^{(+)}e^{-i\omega t}$$

$$x_0^{(+)} = \frac{eE_0^{(+)}/m}{\omega^2 - \omega_0^2 + i\gamma\omega}$$

On resonance there is a phase lag of π/2 and there is no divergence.

$$\delta = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

Polarizability (scalar):

$$\alpha(\omega) = \frac{e^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Suceptibility (scalar)

$$\chi(\omega) = \frac{Ne^2/m\epsilon_0}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Relation between the polarizability and the dipoles

$$\mathbf{d}^{(+)} = \alpha(\omega)\mathbf{E}^{(+)} \text{ and } \mathbf{P}^{(+)} = \epsilon_0\chi\mathbf{E}^{(+)}$$

Complex refraction index

$$\tilde{n}(\omega) = \sqrt{1+\chi(\omega)} \approx 1 + \frac{\chi(\omega)}{2}$$

$$\tilde{n}(\omega) \approx 1 + \frac{Ne^2}{2m\epsilon_0}\frac{(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} + i\frac{Ne^2}{2m\epsilon_0}\frac{\gamma\omega}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}$$

$$E(z) = E_0 \exp(ikz) = E_0 \exp(i\tilde{n}k_0 z)$$

$$= E_0 \exp(i\mathrm{Re}[\tilde{n}]k_0 z)\exp(-\mathrm{Im}[\tilde{n}]k_0 z)$$

Index of refraction and absorption coefficient

$$n(\omega) := \mathrm{Re}[\tilde{n}(\omega)]$$

$$a(\omega) := 2k_0\mathrm{Im}[\tilde{n}(\omega)]$$

Near resonance $|\omega - \omega_0| \ll \omega_0$

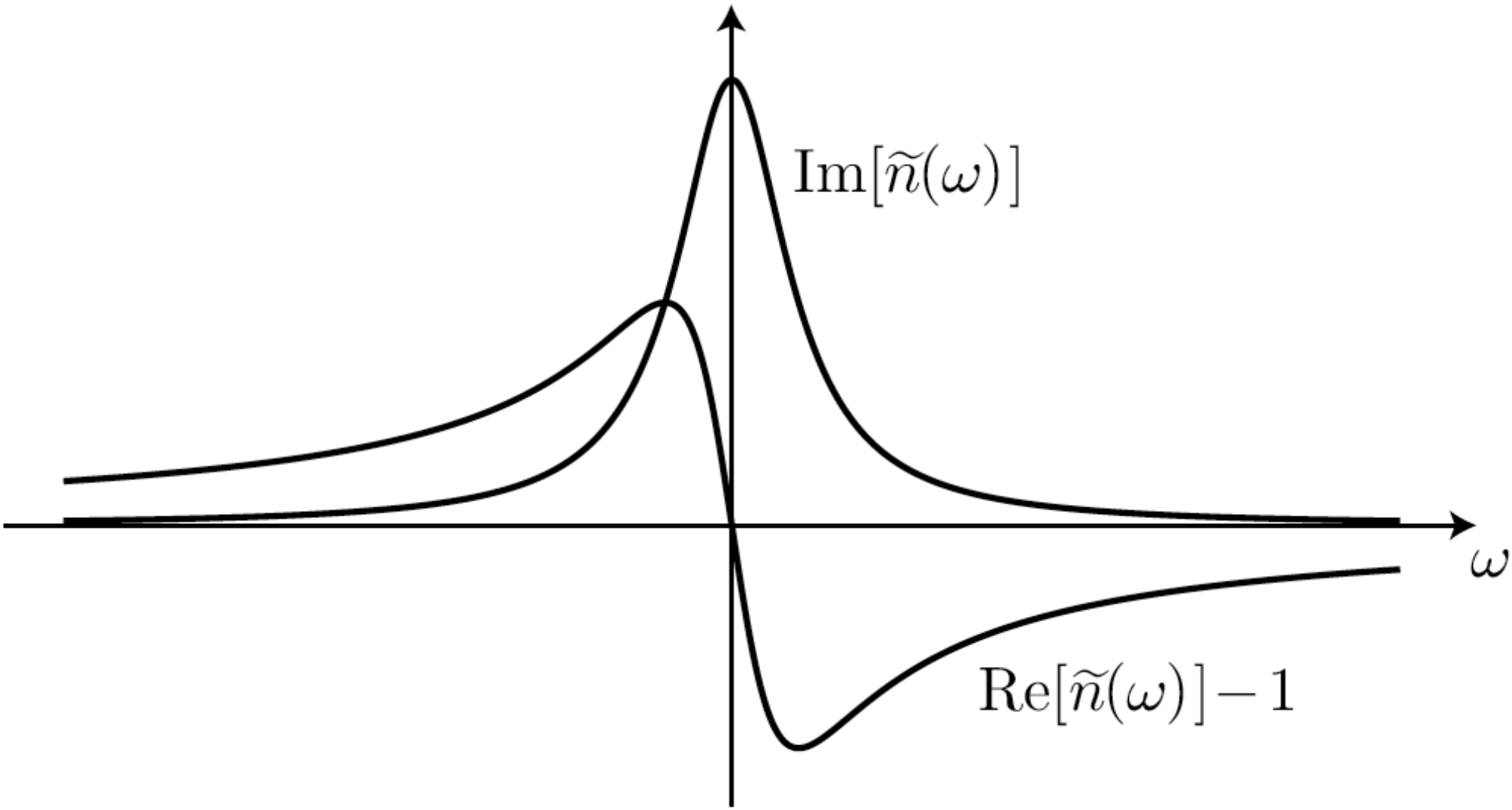
$$n(\omega) \approx 1 + \frac{Ne^2}{2m\epsilon_0} \frac{(\omega_0 - \omega)/2\omega}{(\omega_0 - \omega)^2 + (\gamma/2)^2}$$

$$a(\omega) \approx \frac{Ne^2}{m\epsilon_0 c\gamma} \frac{(\gamma/2)^2}{(\omega_0 - \omega)^2 + (\gamma/2)^2}.$$

Lorenzian lineshape (one derivative-like of the other).

Related to the Kramers Kronig relations.

Lorentzian approximiation near resonance:



Beer's Law for the tranmission of light through an absorbing medium

$$\frac{dI}{dz} = -aI \quad \Longrightarrow \quad I(z) = I_0 e^{-az}$$

$$a(\omega_0) = \sigma(\omega_0)N = \sigma_0 N$$

$$\sigma_{\text{classical}}(\omega_0) = \frac{e^2\omega^2}{m\epsilon_0 c}\frac{\gamma}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}\bigg|_{\omega=\omega_0} = \frac{e^2}{m\epsilon_0 c\gamma}$$

Abrahams Lorentz model:

$$\gamma = \frac{e^2\omega_0^2}{6\pi m\epsilon_0 c^3}$$

The result for the classical radiation cross section of a dipole gives:

$$\sigma = \frac{3\lambda^2}{2\pi}$$

Coincides with the result from quantum mechanics for a two level atom

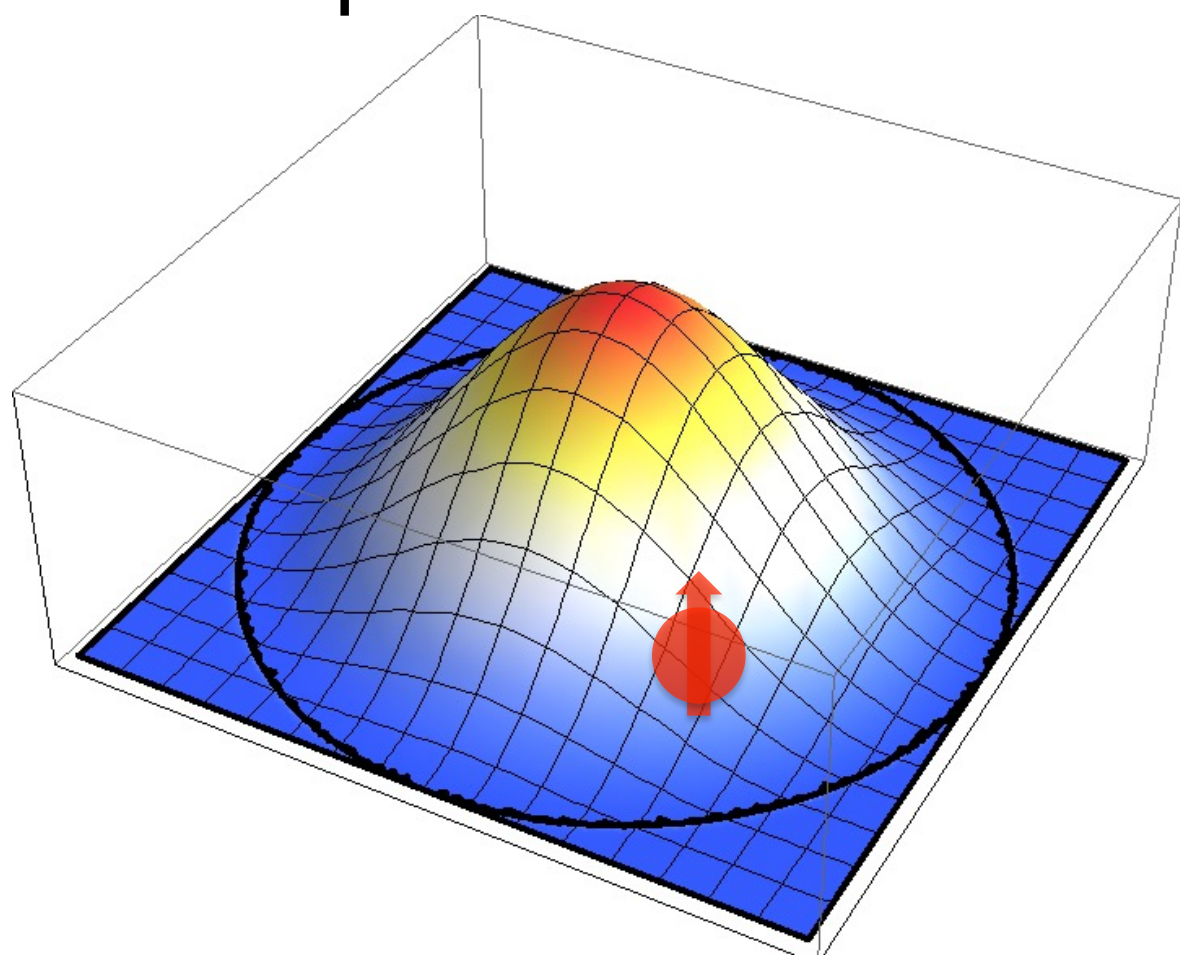
3. Atom field coupling

Normal Coupling

The "absorption" of classical dipole is its cross section: $\sigma = 3\lambda^2/2\pi$ (same as a QM two level atom).

The energy of an electric dipole d in an electric field E:

$$H_{\text{int}} = \vec{d} \bullet \vec{E}$$



: atom, dipole

Rate of decay (Fermi's golden rule)

$$\gamma_{rad} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Phase space density

Interaction

What is the mode density?

The phase space where the emitted light or particles have to “land”

What is the interaction?

This could be electric, magnetic, weak, or strong.

The spectral mode density for two polarizations n(ω)

$$dk_x dk_y dk_z = 2 \times 4\pi k^2 dk$$

$$n(\omega) d\omega = 2 \times 4\pi \frac{\omega^2}{c^3} d\omega$$

$$n(\omega) = 8\pi \frac{\omega^2}{c^3}$$

The QM electric dipole interaction, operators:

$$H_{\text{int}} = \vec{d} \bullet \vec{E}$$

QM Operator :

$$\vec{d} = e \left\langle \Psi_i \middle| \vec{r} \middle| \Psi_f \right\rangle$$

Note that this integral is zero if the two states have the same parity, for example if they are the same: when the atom is in the ground state or when it is in the excited state!

Rate of decay (Fermi's golden rule)

$$\gamma = \frac{\omega_0^3 d^2}{\pi \varepsilon_0 \hbar c^3}$$

Decay into the nanofiber mode

Density of modes in 1D

$$\gamma_{1D} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Decay into the nanofiber mode

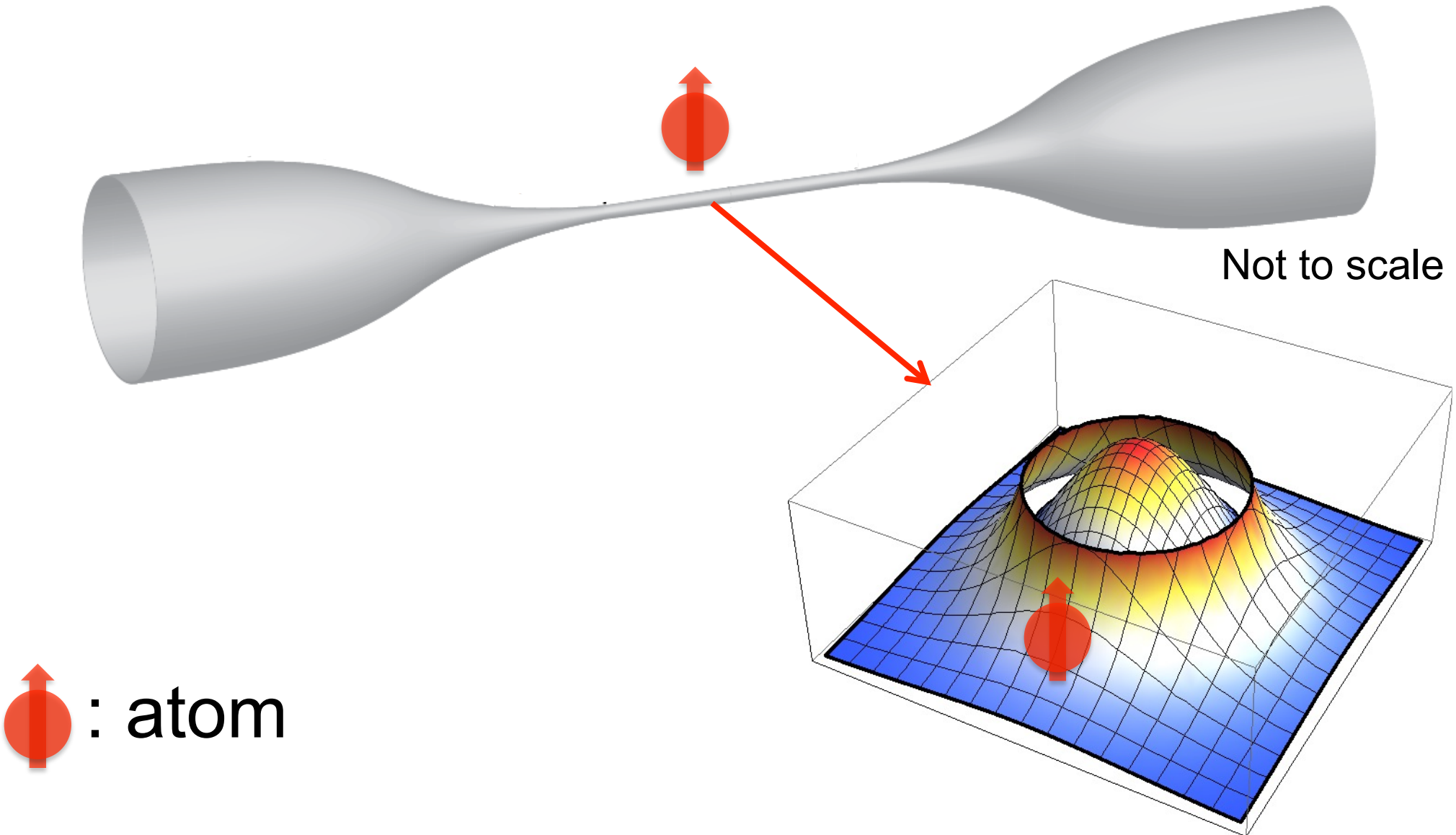
Density of modes

$$\gamma_{1D} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Proportional to the electric field of the guided mode.

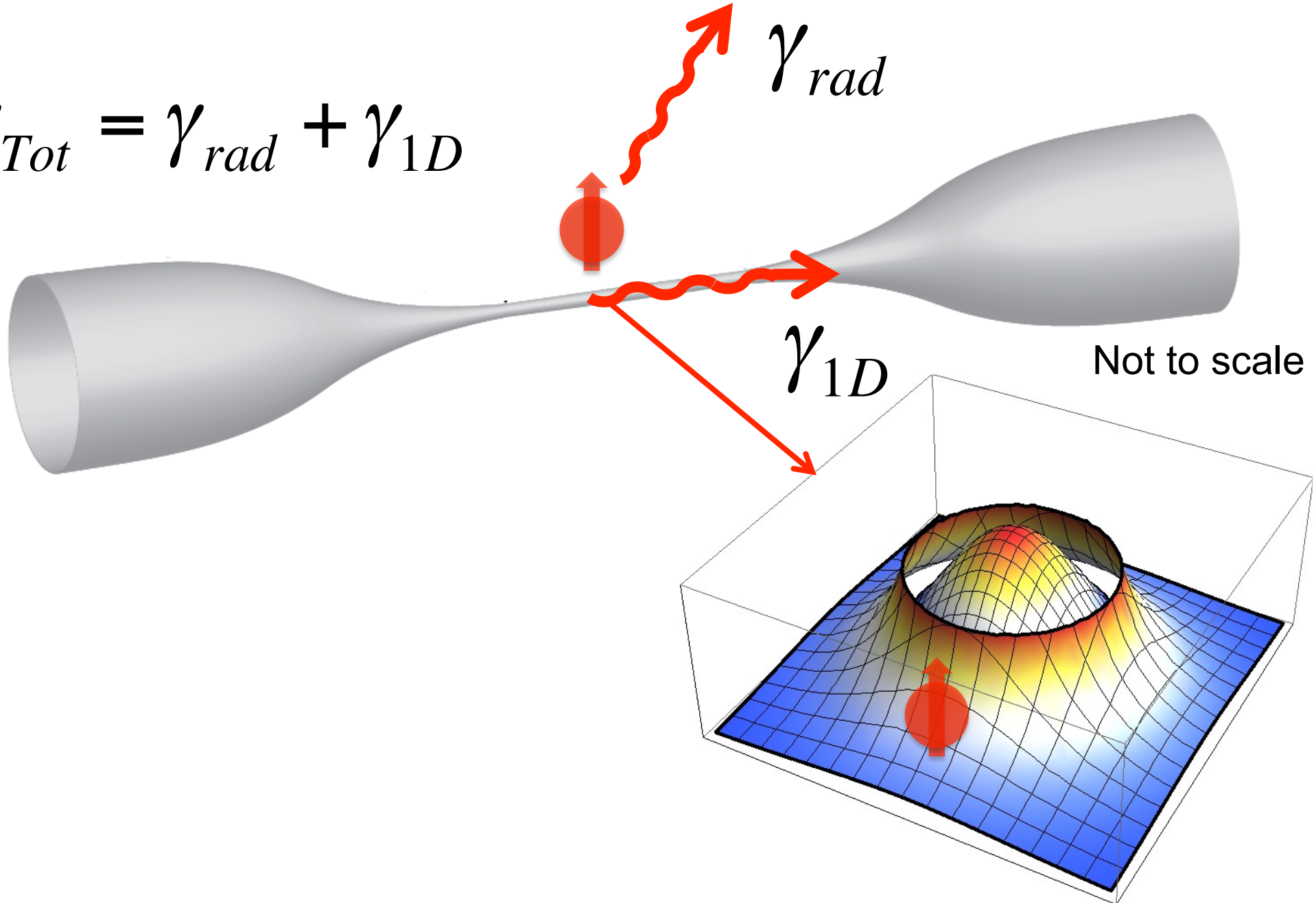
$$|E|^2 = \mathcal{E}^2 \left[K_0^2(qr) + w K_1^2(qr) + f K_2^2(qr) \right]$$

Evanescent Coupling



Evanescent Coupling

$$\gamma_{Tot} = \gamma_{rad} + \gamma_{1D}$$



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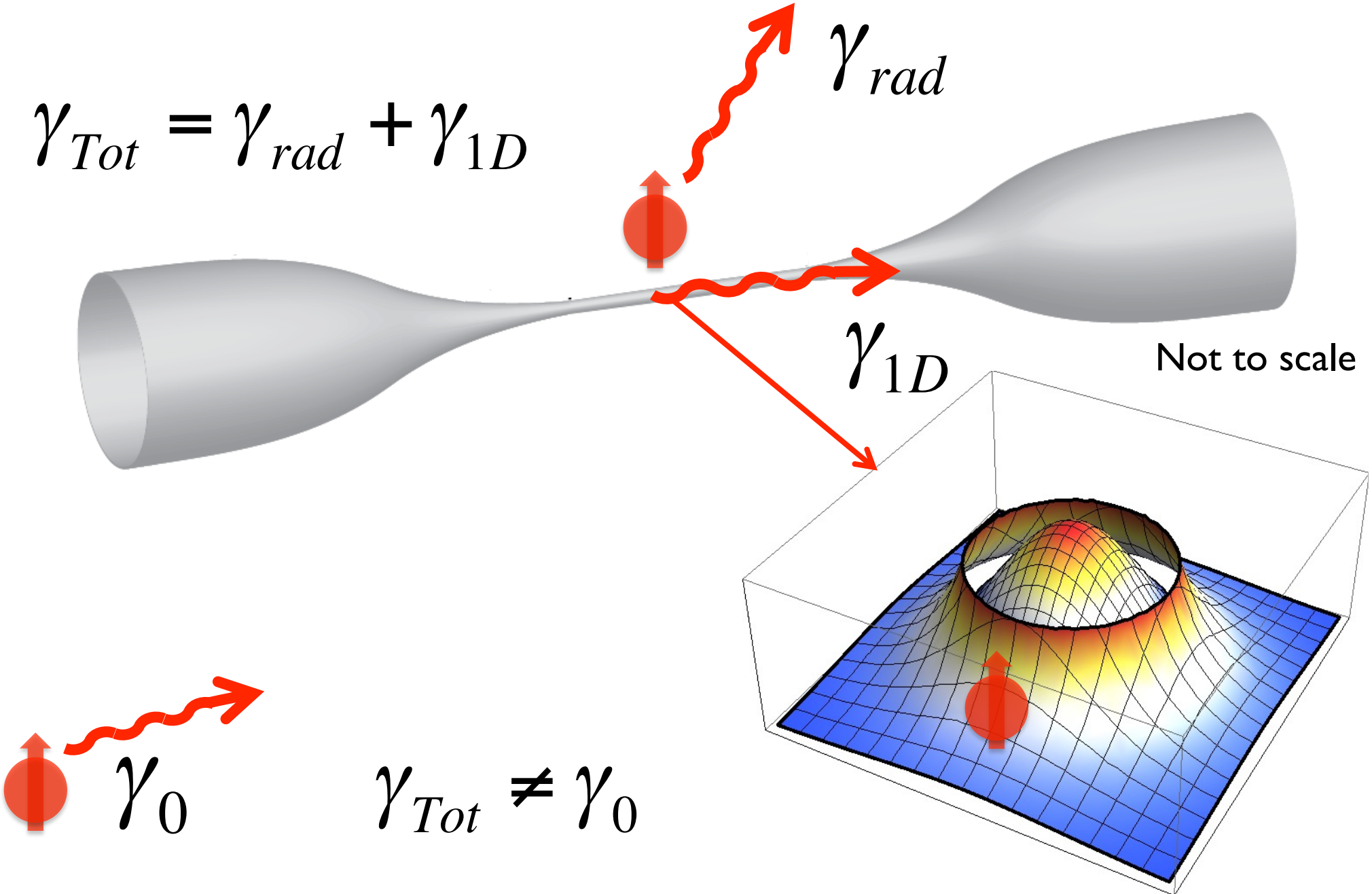
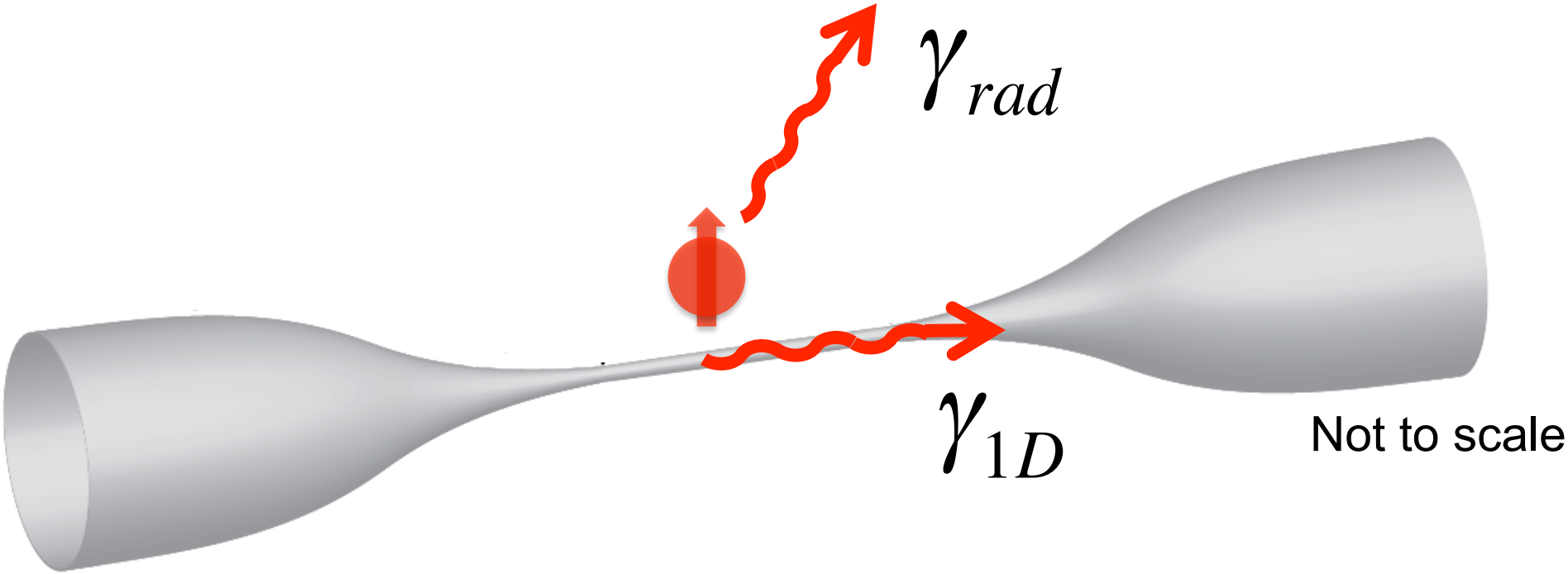


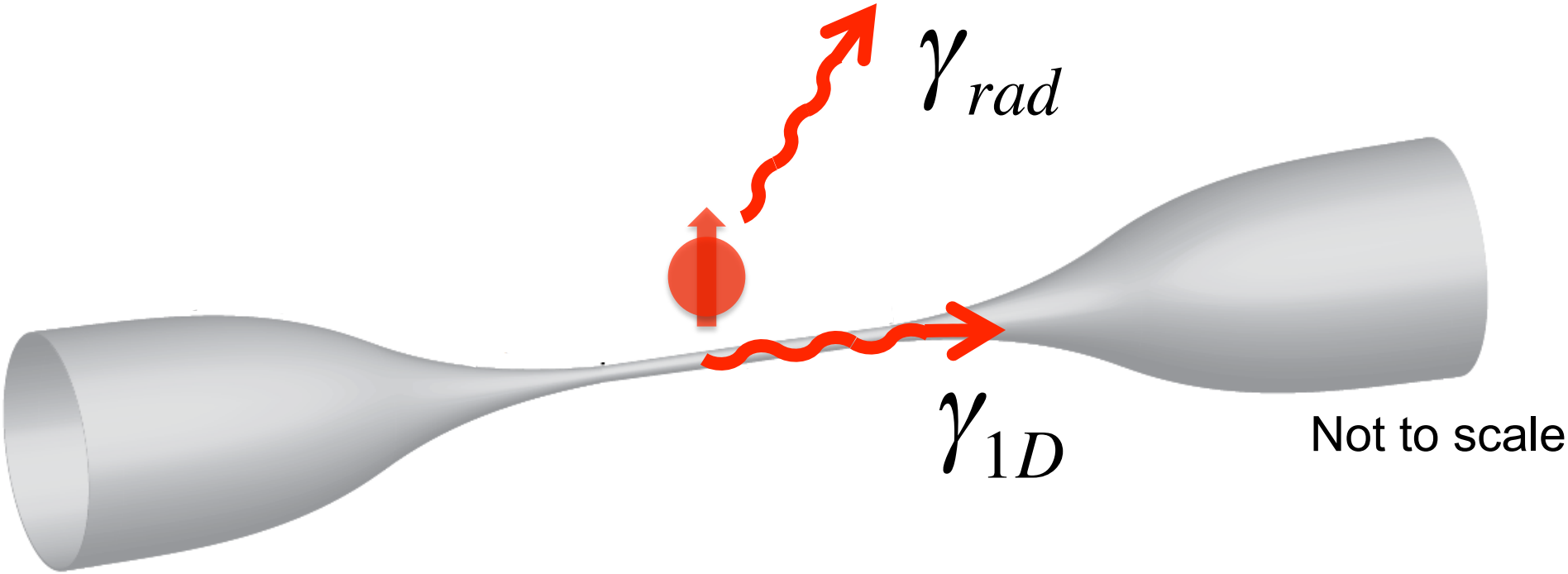
$$\gamma_{Tot} \neq \gamma_0$$

Coupling Enhancement



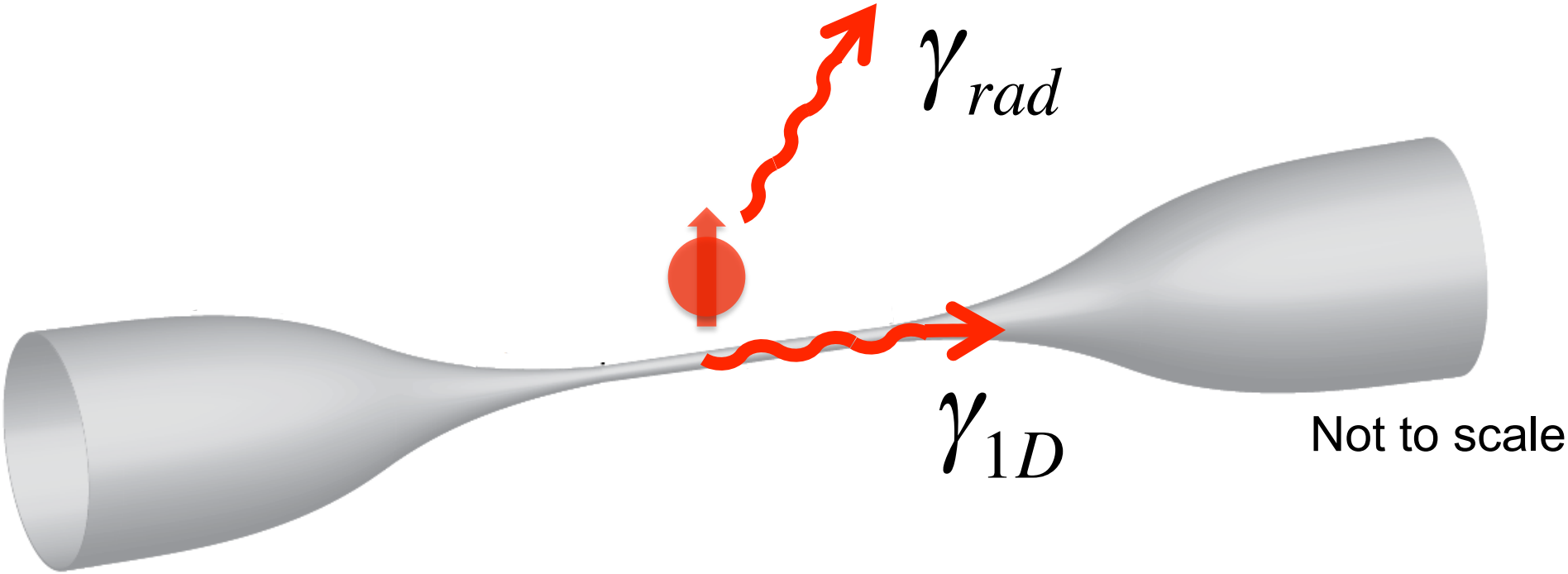
$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

Coupling Efficiency



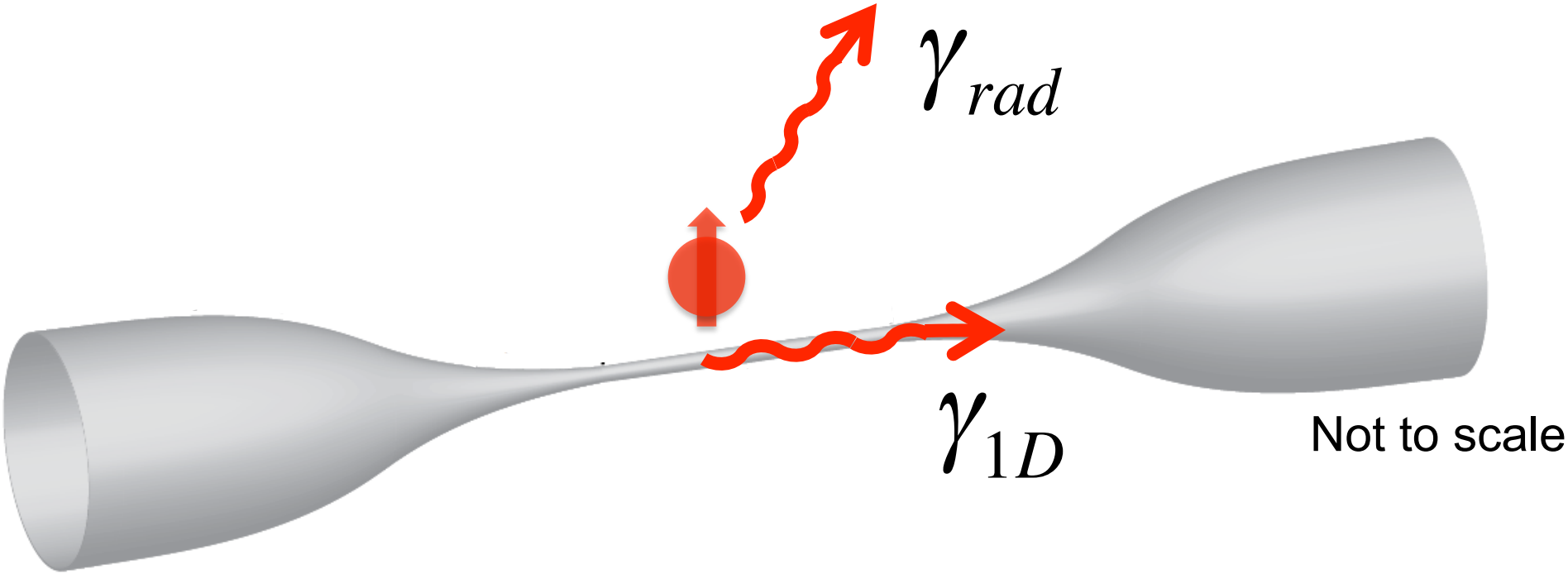
$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}}$$

Cooperativity



$$C_1 = \frac{\beta}{(1-\beta)}$$

Cooperativity



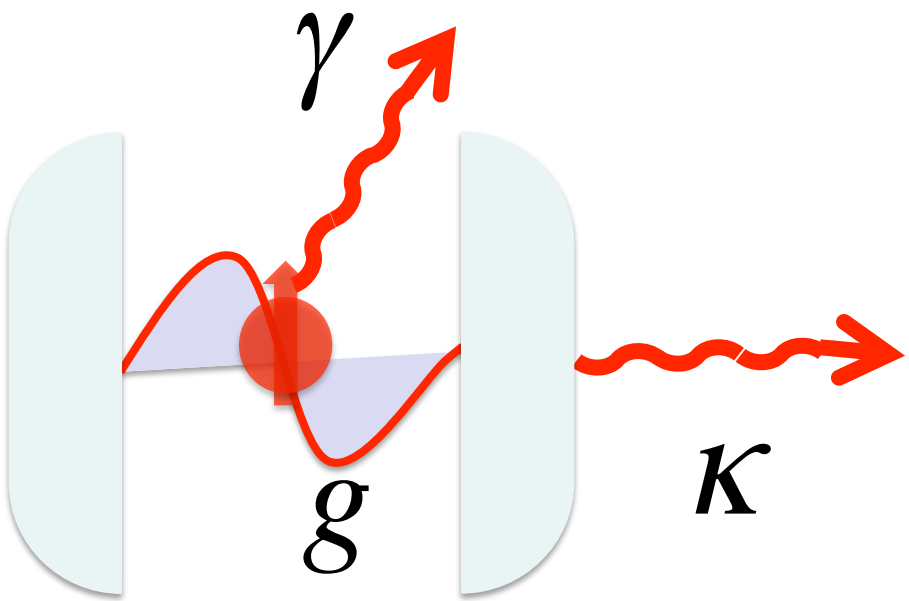
$$C_1 = \frac{\beta}{(1-\beta)}$$

What is the physical meaning!

Cavity QED reminder

$$g = \frac{|\vec{d} \cdot \vec{E}_v|}{\hbar}$$

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$



$$C_1 = \frac{g^2}{\kappa\gamma} \qquad (C_N = NC_1)$$

Cavity QED reminder

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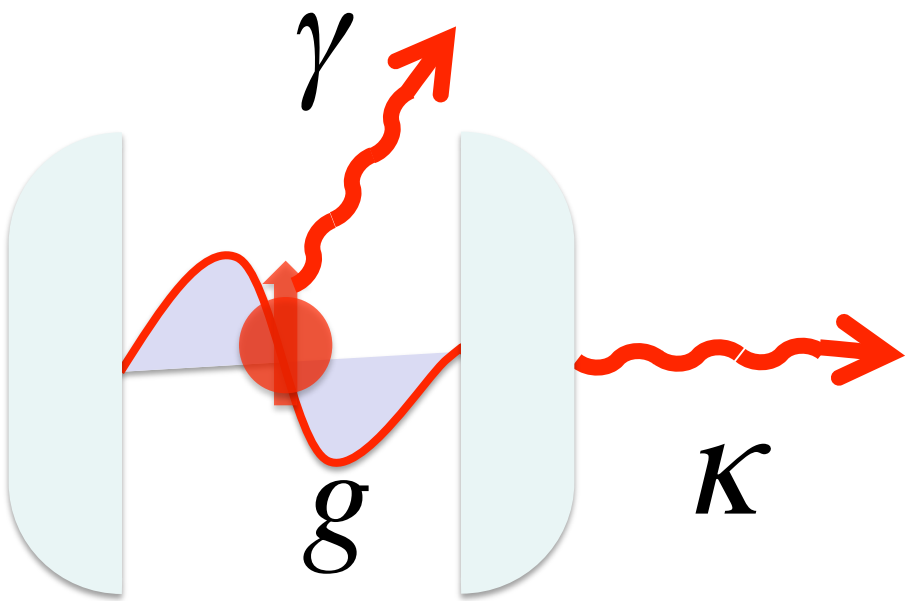


useful interactions

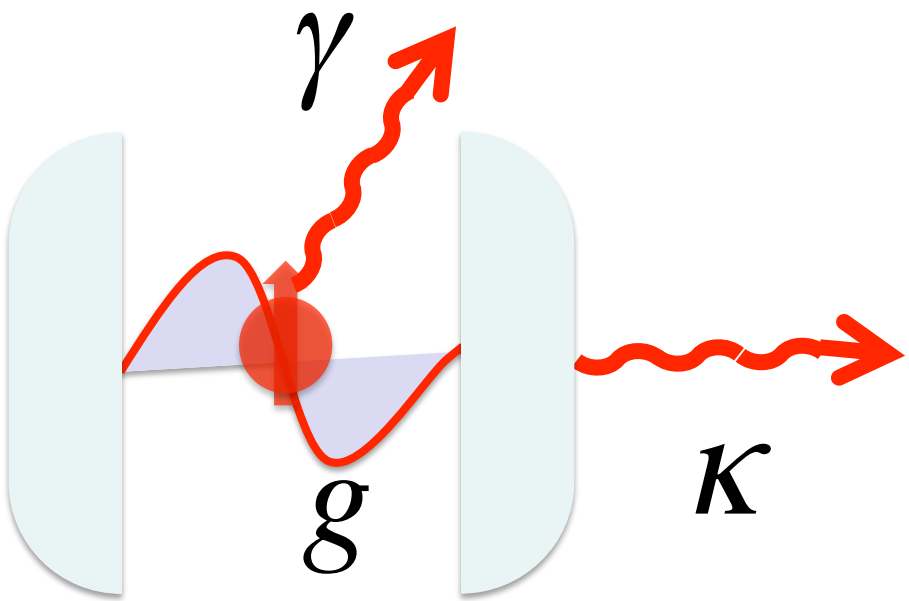
$$C_1 = \frac{g^2}{\kappa\gamma} \qquad (C_N = NC_1)$$

detrimental interactions

Cavity QED reminder

$$g = \frac{|\vec{d} \cdot \vec{E}_v|}{\hbar}$$

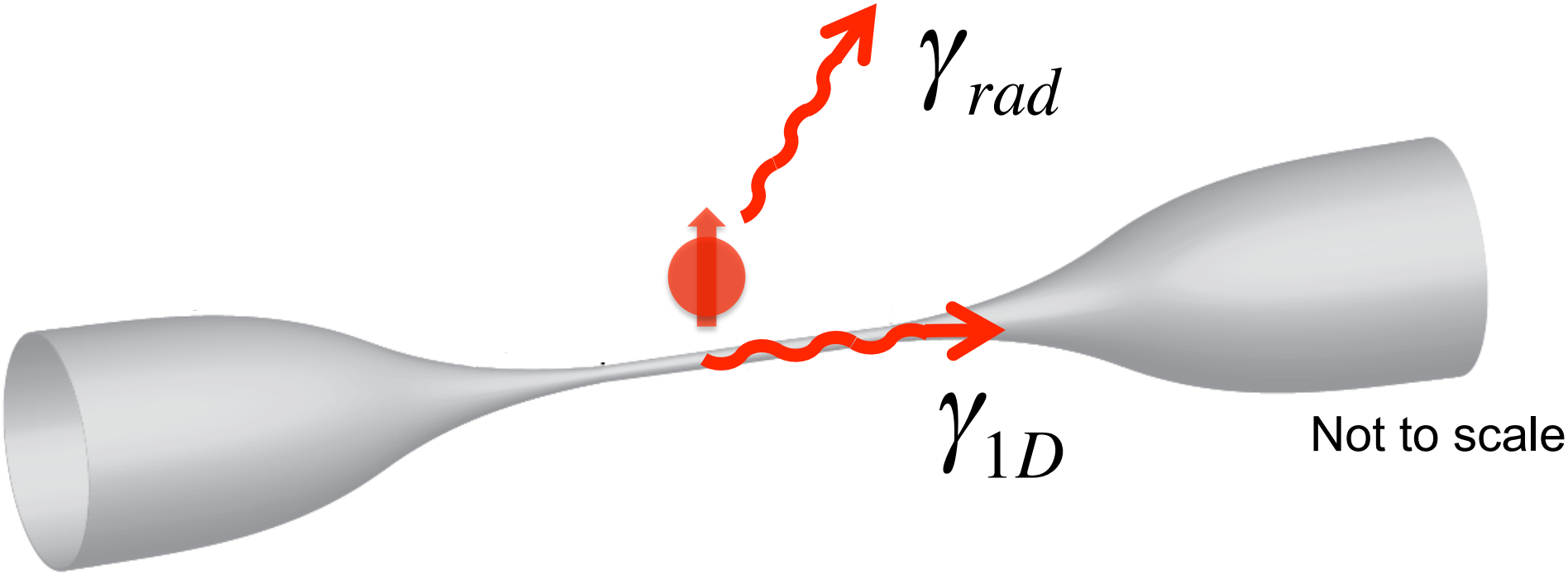
$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$



$$C_1 = \frac{g^2}{\kappa\gamma} \qquad (C_N = NC_1)$$

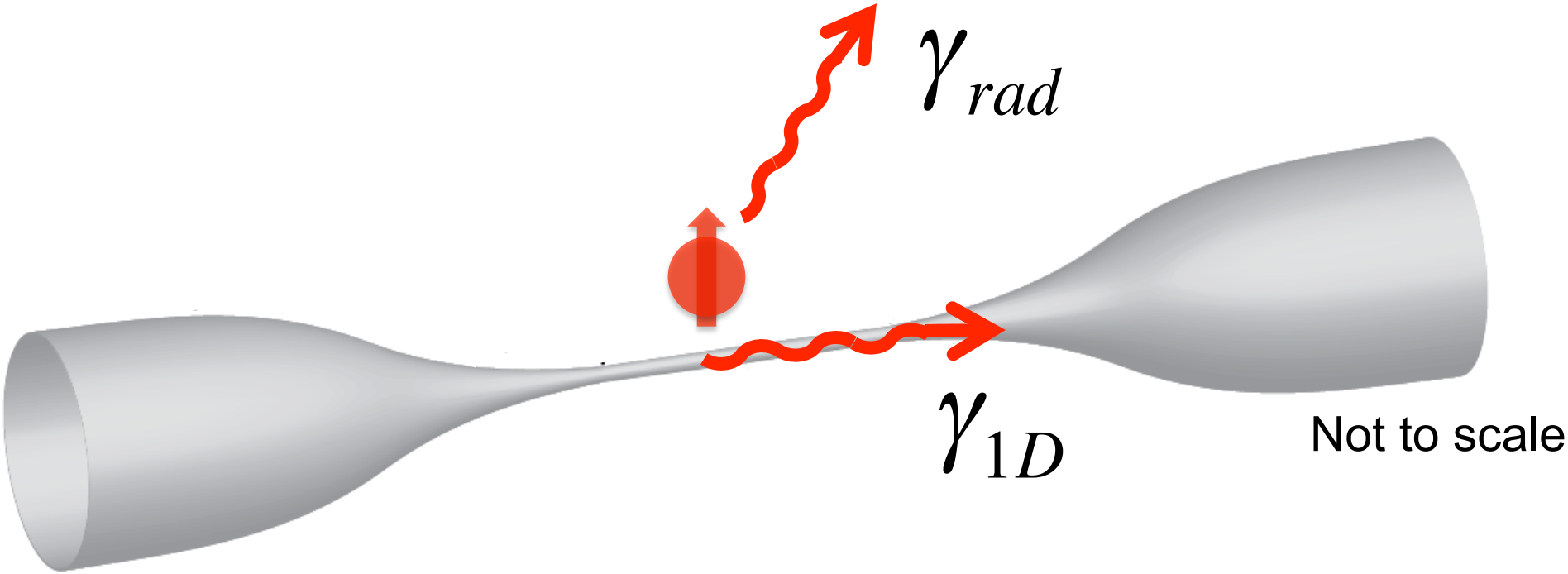
C is the figure of merit for coherent interactions! Key in quantum information

Cooperativity



$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

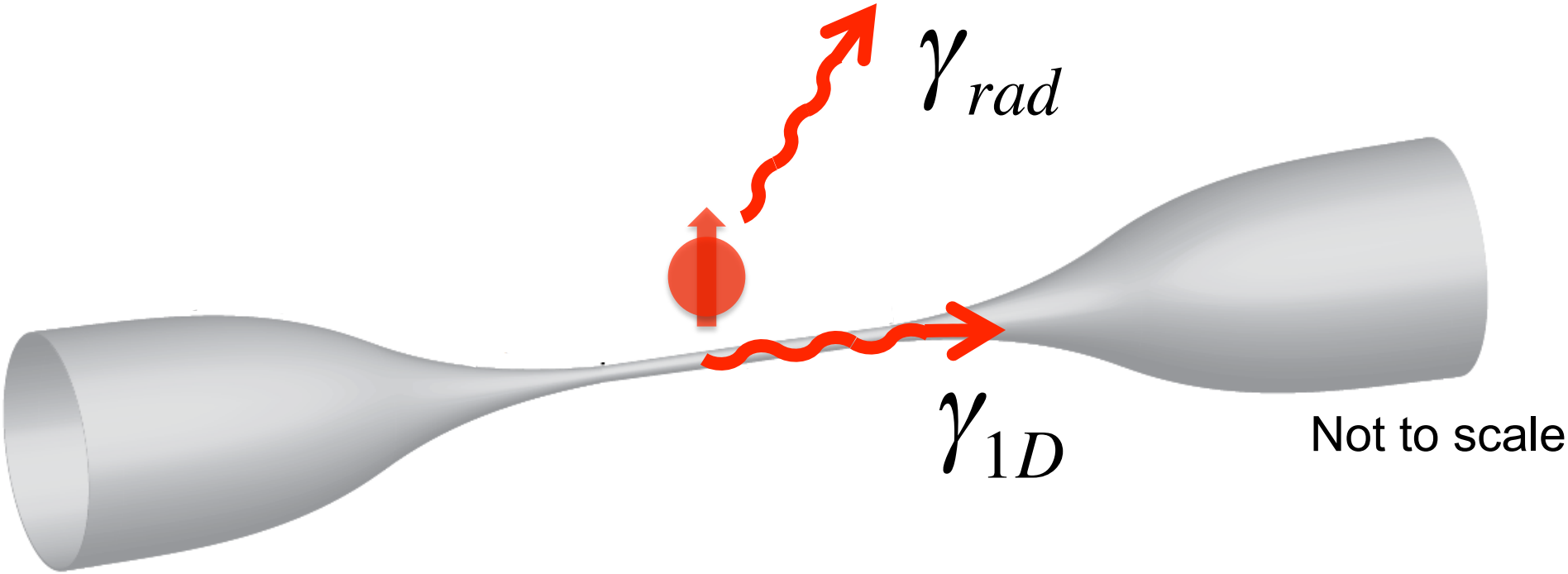
Cooperativity



$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

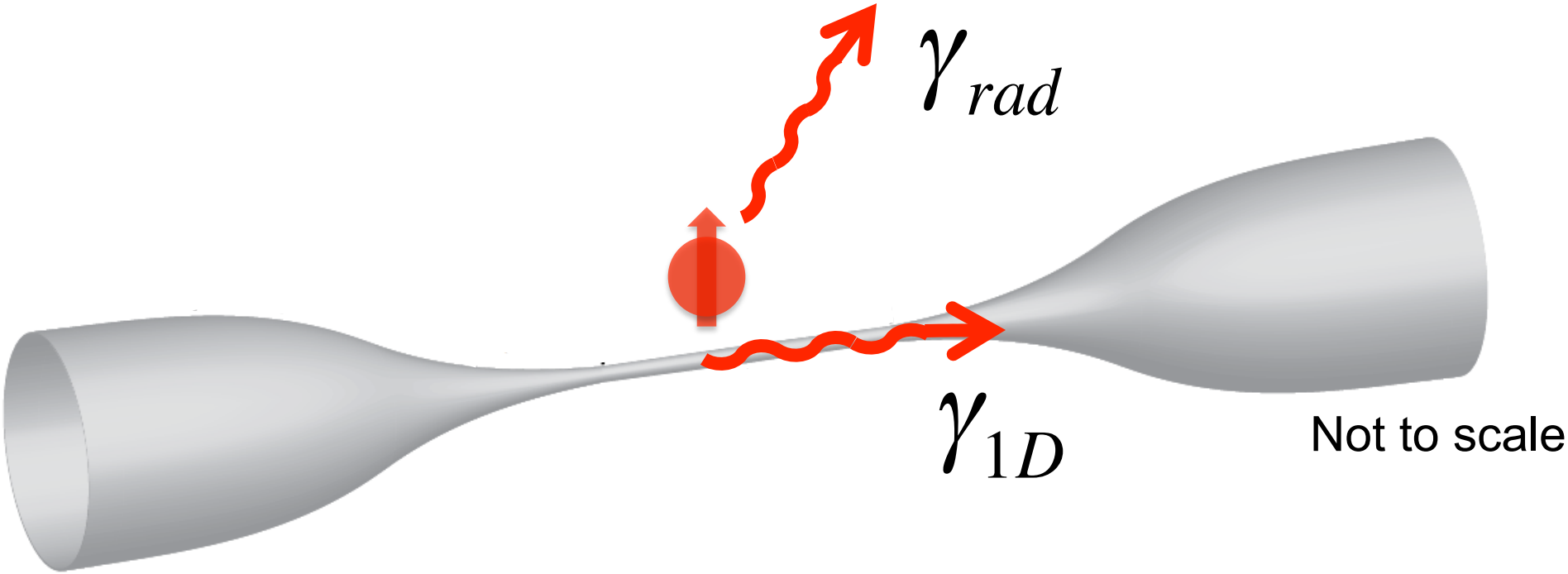
C_1 is the ratio of what goes into the selected mode to what goes into all the rest

Purcell Factor



$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

Purcell Factor

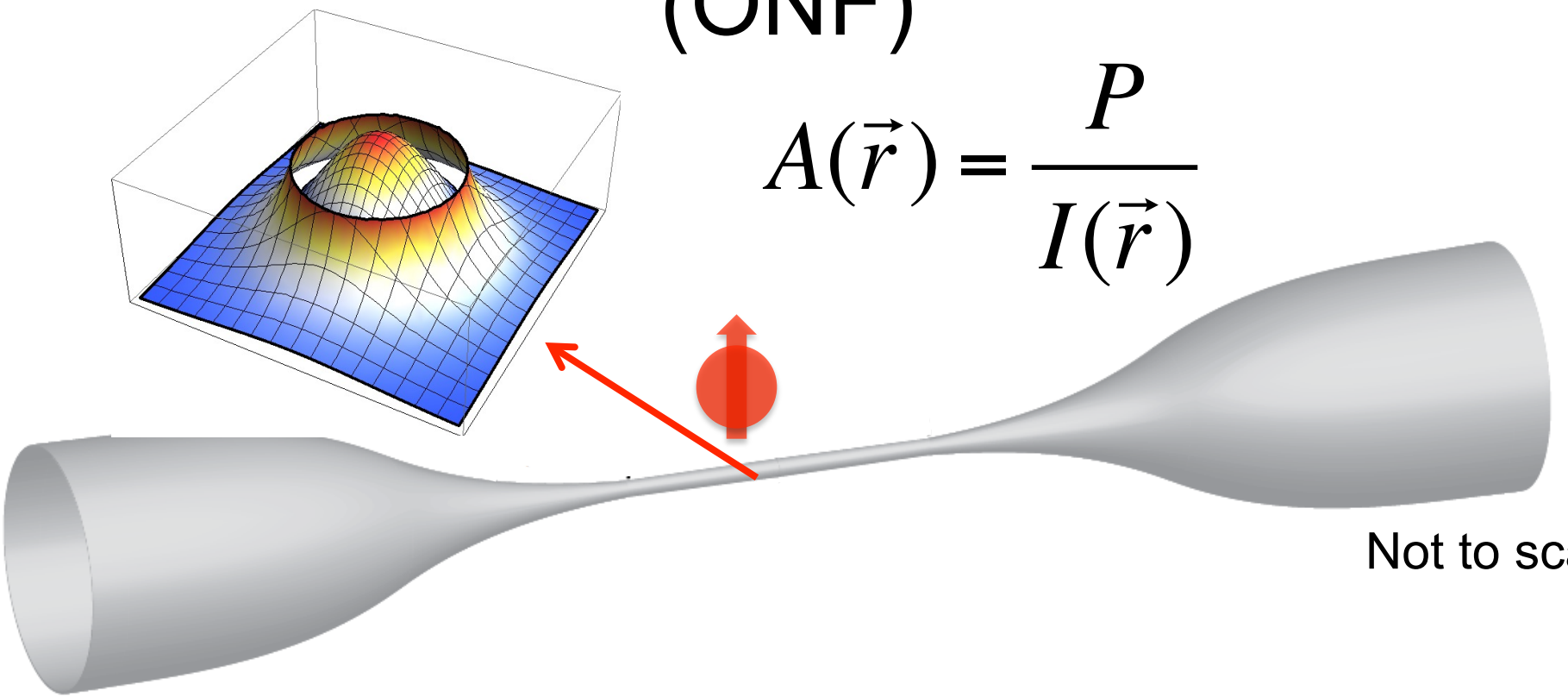


$F_P > 1$ **Enhancement** of spontaneous emission

$F_P < 1$ **Inhibition** of spontaneous emission

Optical Density in an optical nanofiber (ONF)

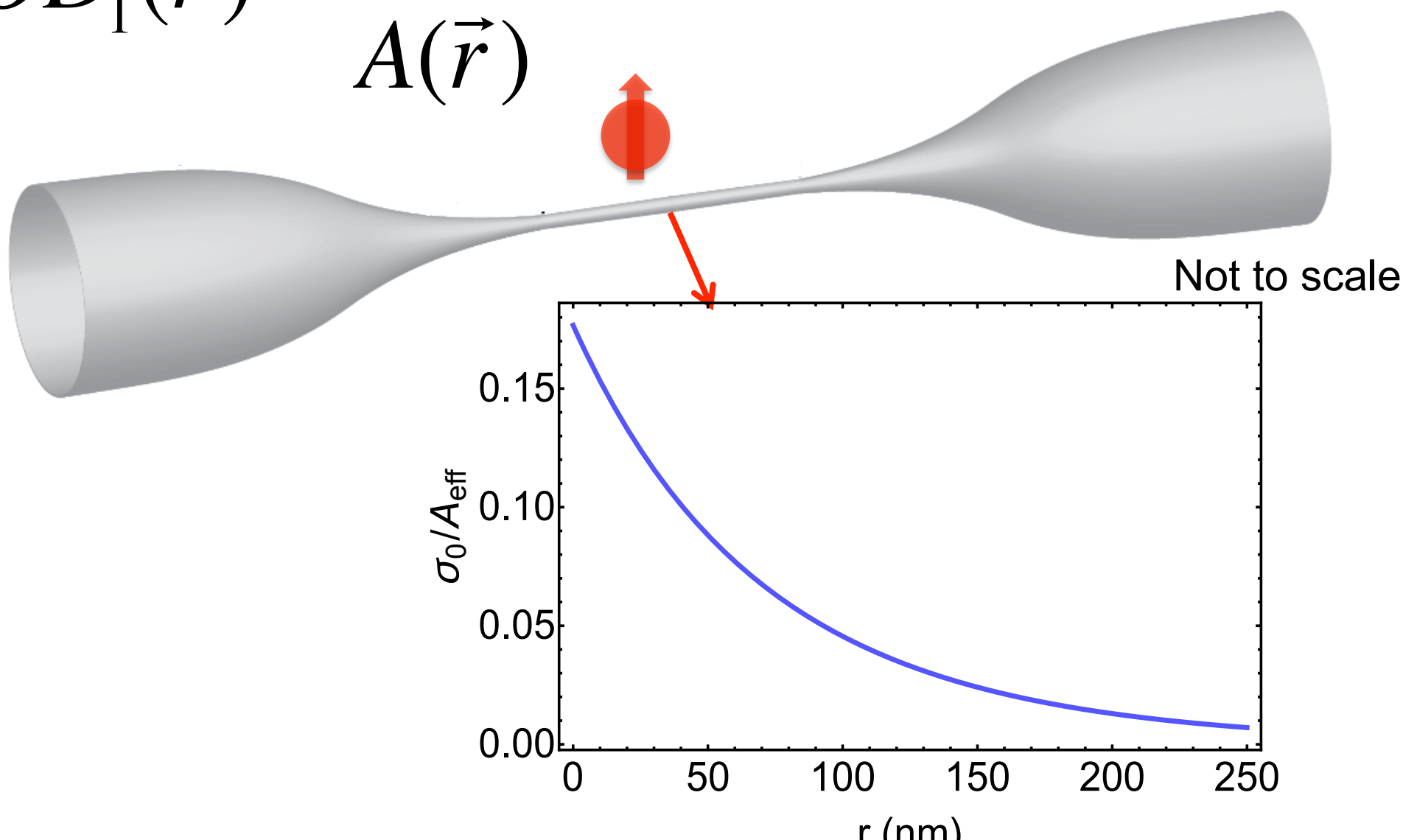
$$A(\vec{r}) = \frac{P}{I(\vec{r})}$$



$$\alpha(\vec{r}) = \frac{\gamma_{1D}(\vec{r})}{\gamma_0} = \frac{1}{n_{eff}} \frac{\sigma_0}{A(\vec{r})} = \frac{1}{n_{eff}} OD_1(\vec{r})$$

ONF Optical Density

$$OD_1(\vec{r}) = \frac{\sigma_0}{A(\vec{r})}$$



ONF Optical Density

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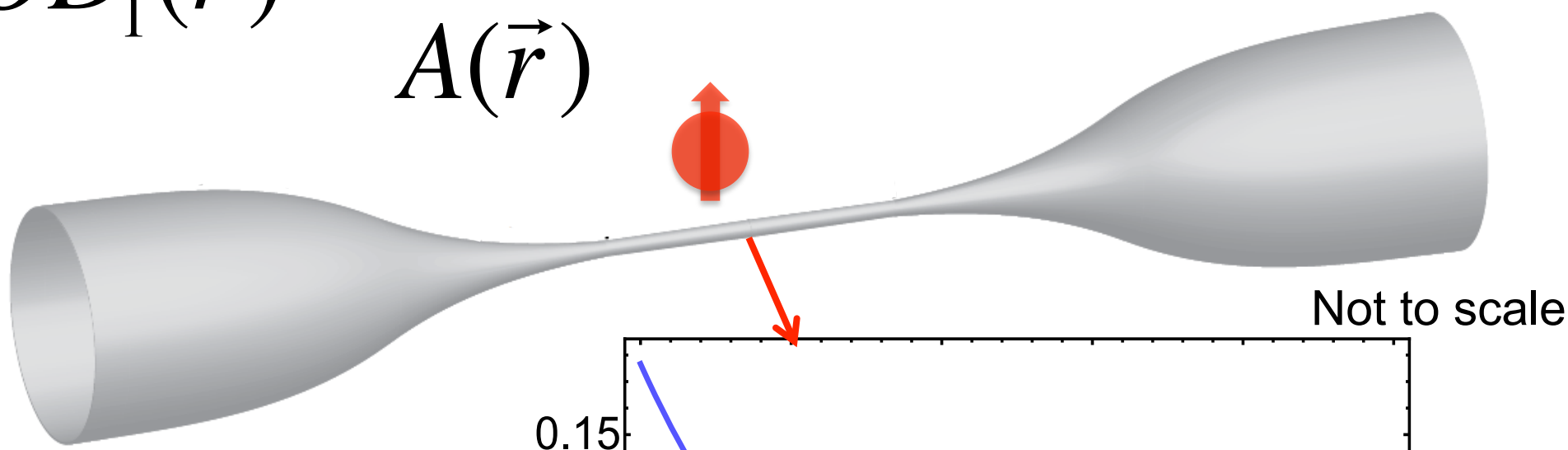
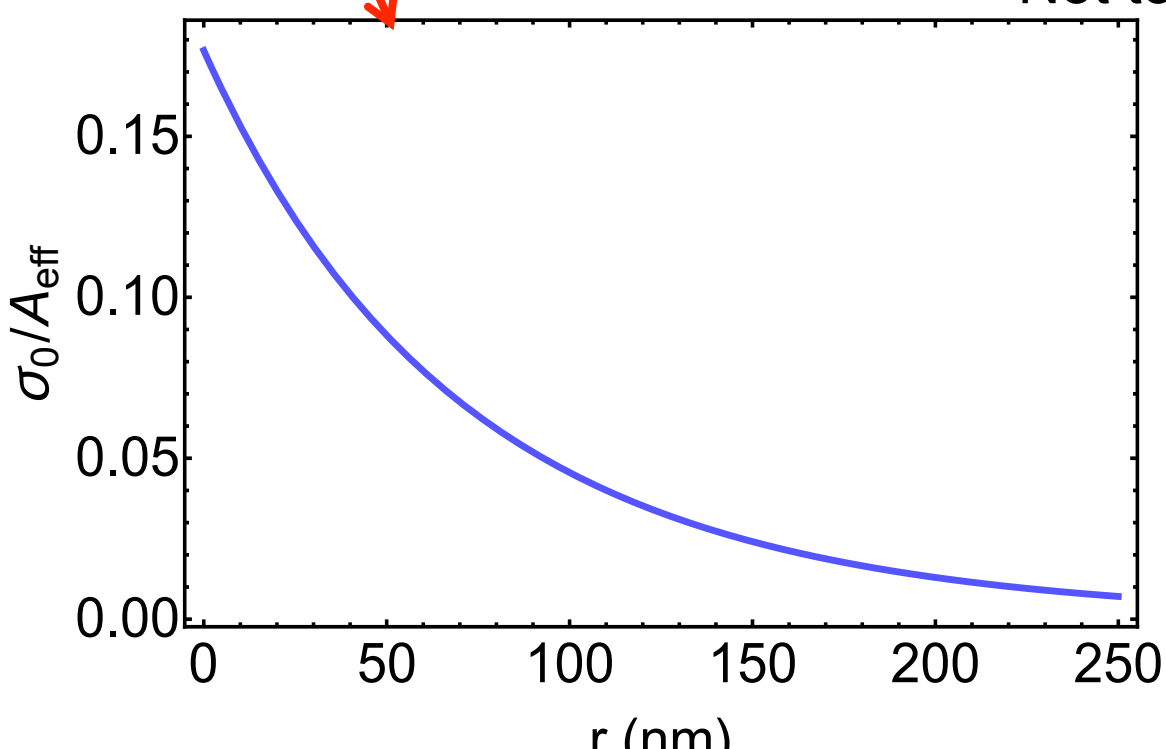
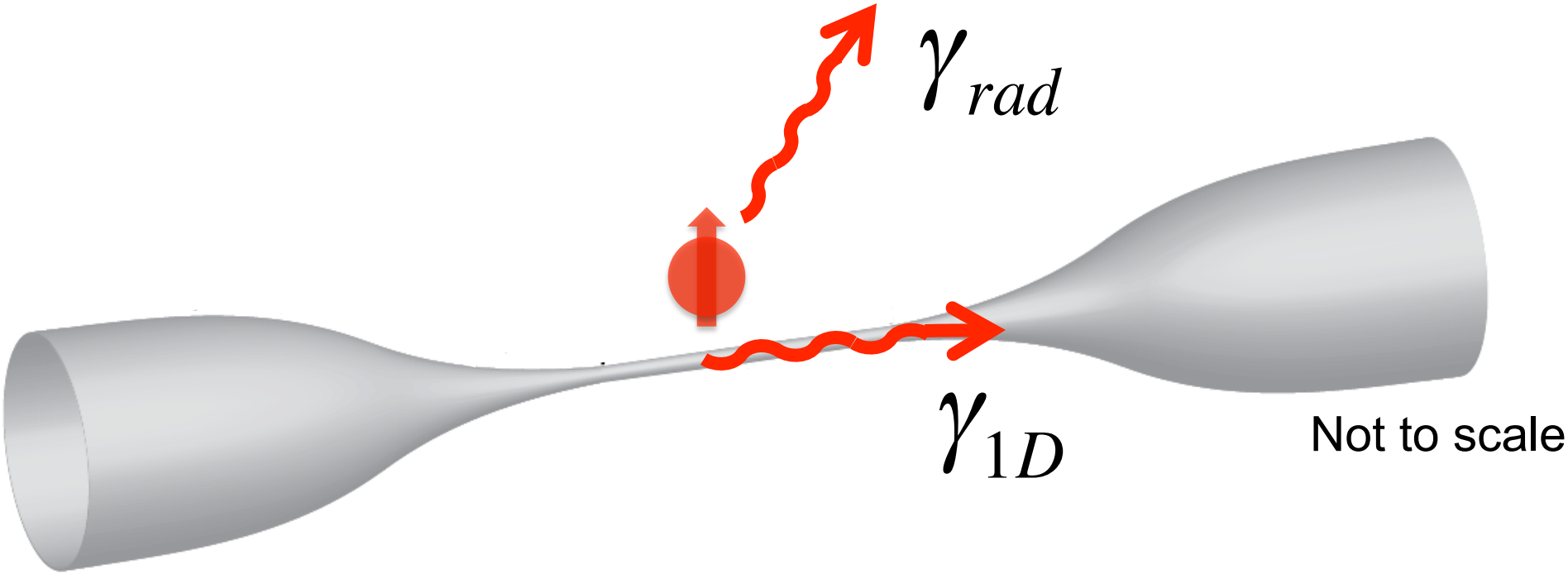


Not to scale



50nm away from the surface
1 atom can block 10% of the light!!

Cooperativity and Optical Density

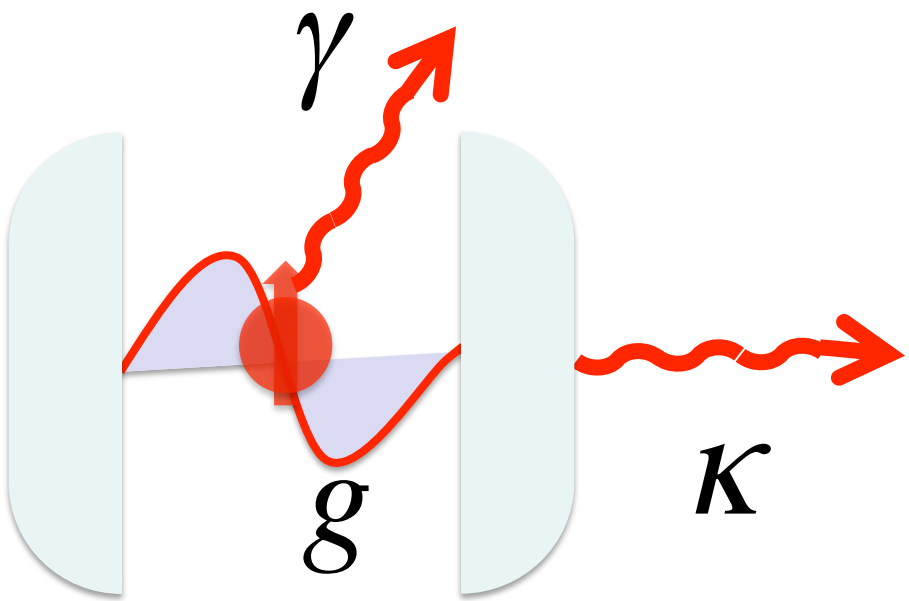


$$C_1 = OD_1 \frac{1}{n_{eff}} \frac{\gamma_0}{\gamma_{rad}}$$

Cavity QED reminder

$$g = \frac{|\vec{d} \cdot \vec{E}_v|}{\hbar}$$

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

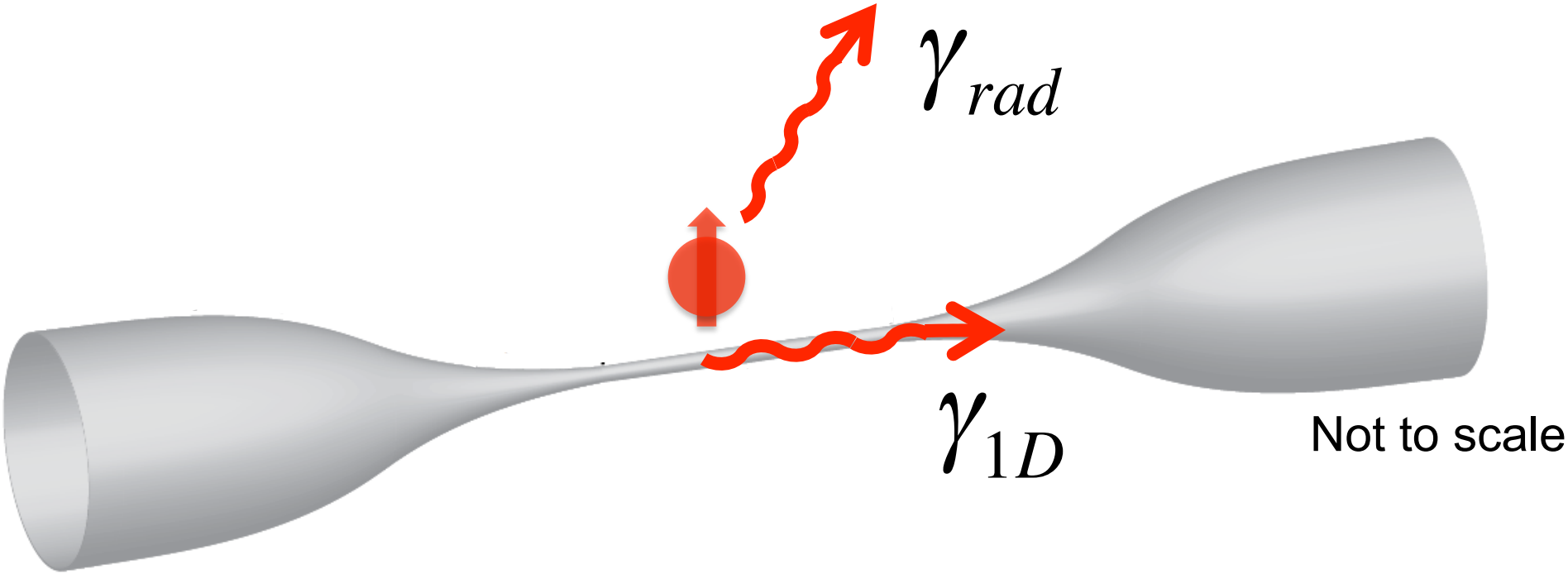


$$C_1 = OD_1 \frac{1}{T} \qquad (T \propto \kappa)$$

For mirrors with transmission T

Enhancement

Cooperativity and Optical Density



$$C_1 = OD_1 \frac{1}{n_{eff}} \frac{\gamma_0}{\gamma_{rad}}$$

Enhancement

Summary:

1. Review of Electricity and Magnetism. Polarization of the Electromagnetic field.
2. Dipoles (antennas, atoms). The model of Lorentz and its response (Polarizability).
3. Different ways to quantify the atom-light coupling with respect to nanofibers and cavity QED.

Gracias