

Classical and quantum properties of vector beams

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Angular momentum of light can be separated into two forms

$$j = (l \pm \sigma)\hbar$$

Spin Angular Momentum      Orbital Angular Momentum

Vector states of light can be mapped on a higher-order Poincaré sphere

$$|U\rangle = \alpha|-\ell\rangle|L\rangle + \beta|\ell\rangle|R\rangle$$

Phys. Rev. Lett. 107 053601 (2011)  
Hölzl et al. Opt. Express 19, 2528 (2011)

$$|\alpha|^2 + |\beta|^2 = 1$$

Using the *Geometric phase* we can create vector beams

(Anisotropic media)  
"extra" phase delay called **geometric phase**

$$\delta = \left(\frac{n_e + n_o}{2}\right) \frac{2\pi}{\lambda} d \pm 2\Phi$$

Geometric Phase

For example, spin-orbit coupling using a geometric phase element with an azimuthal phase variation

$$\begin{pmatrix} \cos 2q\theta & \sin 2q\theta \\ \sin 2q\theta & -\cos 2q\theta \end{pmatrix} \quad \begin{pmatrix} |\ell, L\rangle \rightarrow |\ell + 2q, R\rangle \\ |\ell, R\rangle \rightarrow |\ell - 2q, L\rangle \end{pmatrix}$$

The diagram illustrates a setup for optical field analysis using a Spatial Light Modulator (SLM). A 'Source' (represented by a black box with a question mark) emits light through two lenses onto an SLM (represented by a grey grid). The SLM modulates the light, which then passes through another lens and is detected by a 'ccd' sensor (represented by a grey box with a question mark). The output is shown as a series of concentric circles, indicating a modulated optical field.

The diagram illustrates the expansion of an unknown field  $U$  into an orthonormal basis of modes. It shows a black square labeled '?' representing the unknown field, which is expanded as follows:

$$U = \sum_{n=0}^{\infty} c_n \Psi_n$$

The expansion is shown as:

$$? = c_1 \begin{matrix} \text{blue square with yellow center} \end{matrix} + c_2 \begin{matrix} \text{blue square with yellow center} \end{matrix} + c_3 \begin{matrix} \text{blue square with yellow center} \end{matrix}$$

Below this, two examples are given:

- $\begin{matrix} \text{blue square with yellow center} \end{matrix} \cdot \begin{matrix} \text{blue square with yellow center} \end{matrix} = 0$
- $\begin{matrix} \text{blue square with yellow center} \end{matrix} \cdot \begin{matrix} \text{blue square with yellow center} \end{matrix} = 1$

Annotations explain the process:

- A green arrow points from the term  $c_n \Psi_n$  to the first mode, with the text "Create these modes".
- A green arrow points from the first mode to the second mode, with the text "Perform this integral".
- A green arrow points from the second mode to the third mode, with the text "Perform this integral".
- A green arrow points from the third mode to the result, with the text "Perform this integral".

We can pass an unknown field through a match filter to find the inner product

SLM

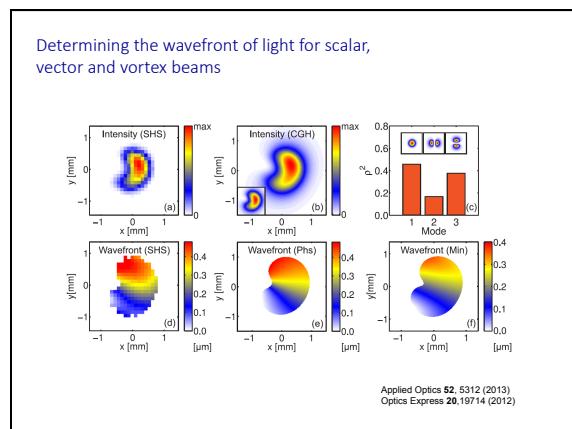
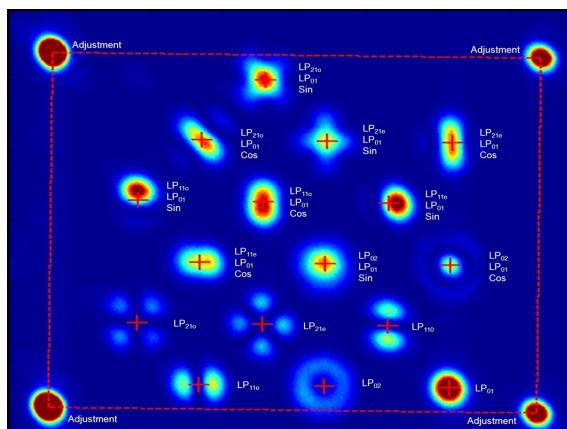
$I = +3$

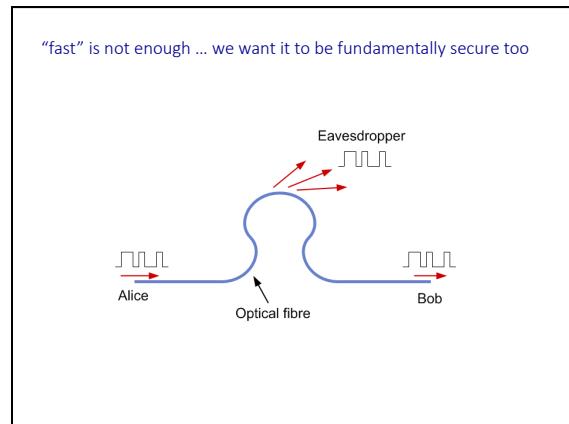
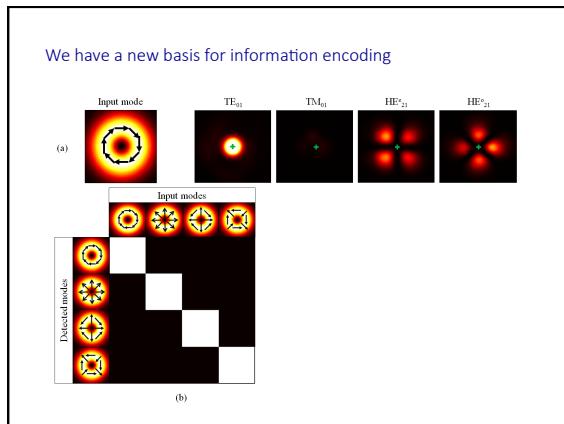
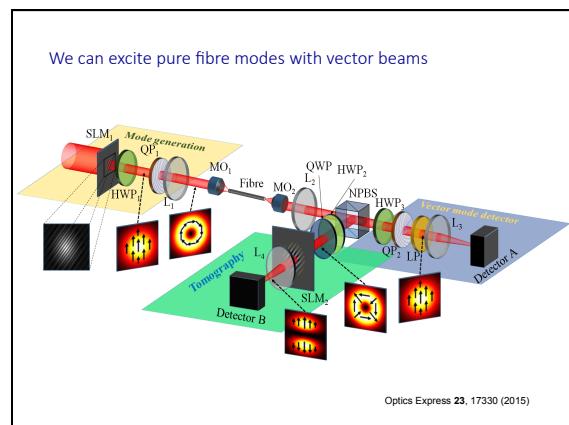
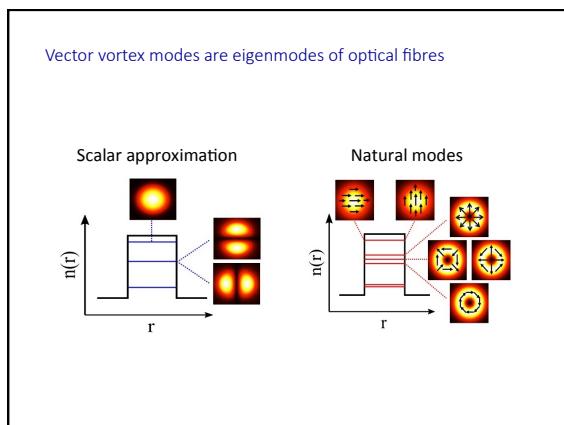
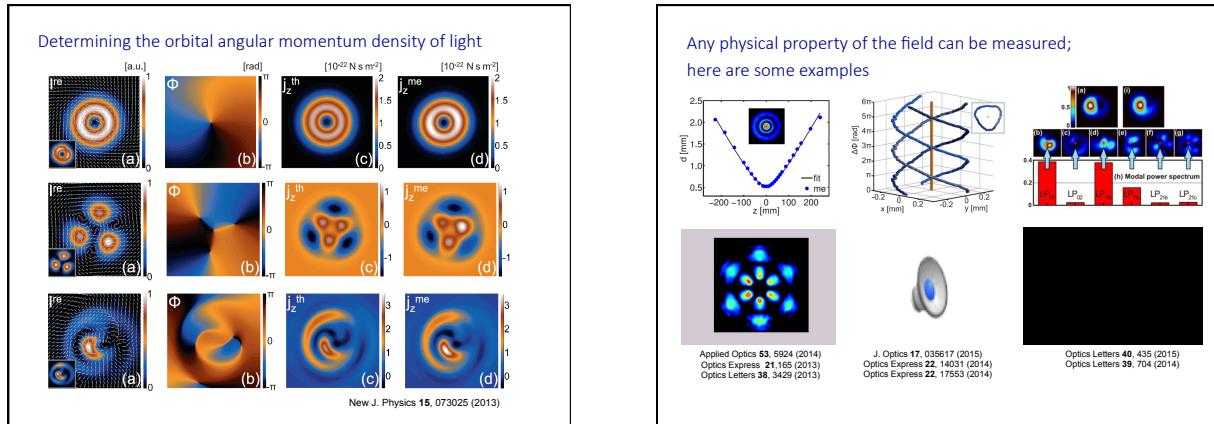
$I = 0$

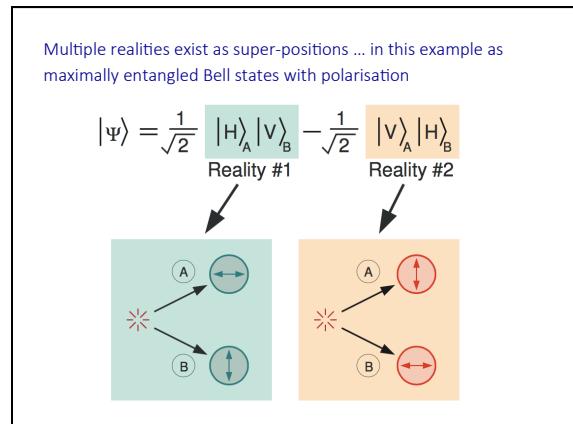
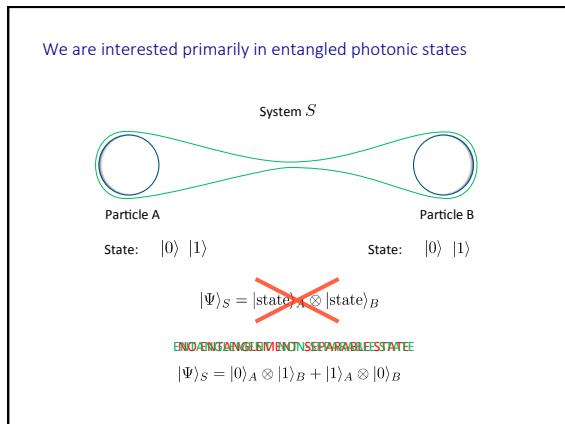
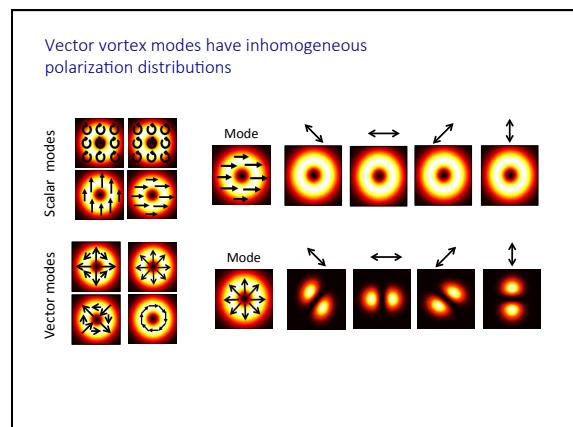
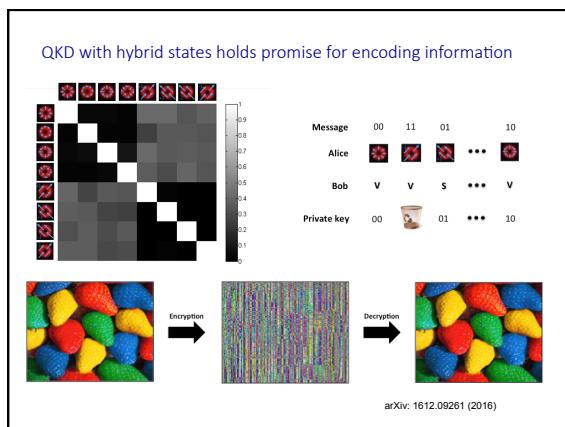
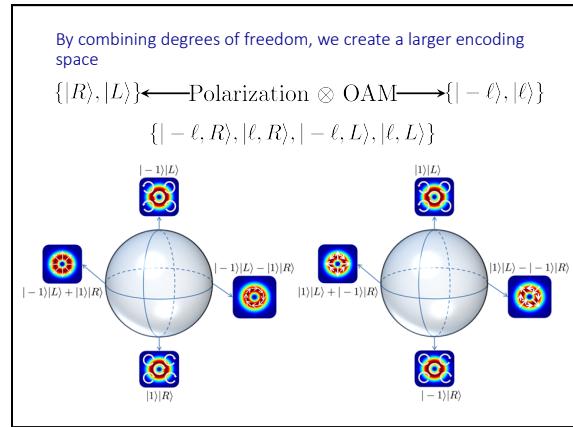
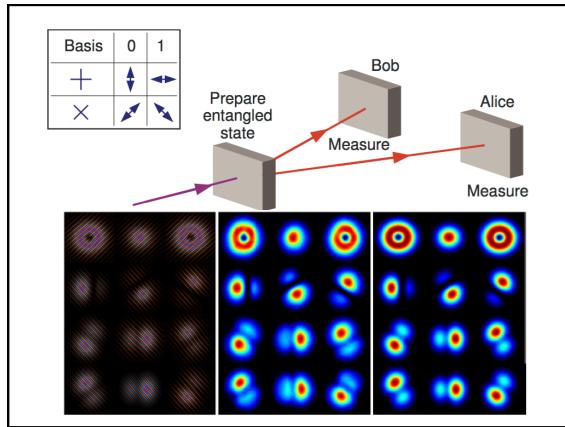
$t^*(r, \theta)$

$U(k_x, k_y) = \iint u t^* \exp(ik_x x + ik_y y) dx dy$

The figure shows two sets of optical modes. On the left, a horizontal double-headed arrow connects two square-shaped modes: one with a central bright spot and a surrounding hexagonal pattern, and another with a central bright spot and a surrounding circular pattern. Above this arrow is the text  $q=1/2$ . Below the first mode is the state  $|0, L\rangle + |0, R\rangle$ . On the right, another horizontal double-headed arrow connects two circular patterns with a star-like interference pattern in the center. Below this second mode is the state  $|0, L\rangle + |0, R\rangle$ .







Entangled particles share information until they are measured

$$|Cat\rangle = |dead\rangle + |alive\rangle$$

Or by “scattering” the entanglement can be destroyed, producing separable states

$$|\Psi\rangle = \frac{1}{2}|H\rangle_A|V\rangle_B - \frac{1}{2}|H\rangle_A|H\rangle_B + \frac{1}{2}|V\rangle_A|V\rangle_B - \frac{1}{2}|V\rangle_A|H\rangle_B$$

... can be factored (separated)

$$|\Psi\rangle = \frac{1}{2}(|H\rangle_A + |V\rangle_A)(|H\rangle_B - |V\rangle_B)$$

$$|\Psi\rangle = \frac{1}{2}\left(\begin{array}{c} \textcirclearrowleft \\ \textcirclearrowright \end{array} \text{ (A)}\right) \left(\begin{array}{c} \textcirclearrowup \\ \textcirclearrowdown \end{array} \text{ (B)}\right)$$

Separability  $\Rightarrow$  Not entangled

Non-separability is not unique to quantum mechanics!

Vector mode

Vector vortex beam

$$|\Psi\rangle = |\ell\rangle_1|R\rangle_2 + |-\ell\rangle_1|L\rangle_2$$

Equivalent?

Quantum entangled state

$$|\Psi\rangle = |\ell\rangle_1|-\ell\rangle_2 + |-\ell\rangle_1|\ell\rangle_2$$

A measurement on one degree of freedom affects the outcome of the other

Doesn't this reminds us of quantum entanglement?

Entanglement:

$$|\Psi\rangle_{AB} = |\ell\rangle_A|-\ell\rangle_B + |-\ell\rangle_A|\ell\rangle_B$$

Vector beams:

$$|\Psi\rangle = |\ell\rangle|R\rangle + |-\ell\rangle|L\rangle$$

Entanglement:

Vector beams:

Measure 1 property

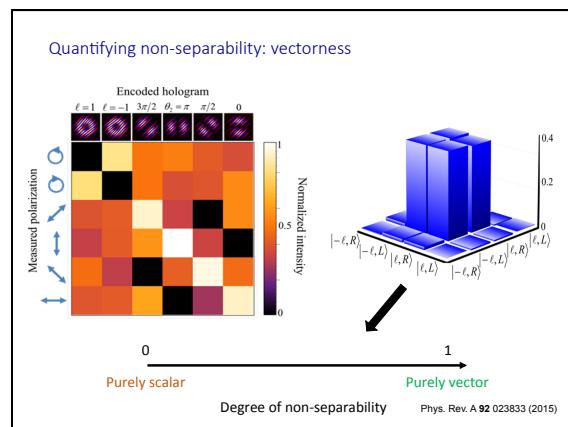
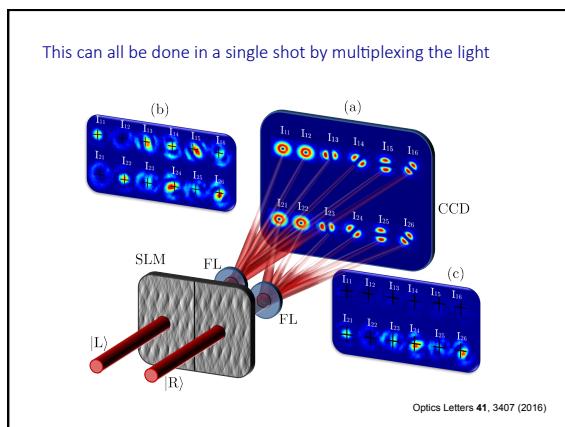
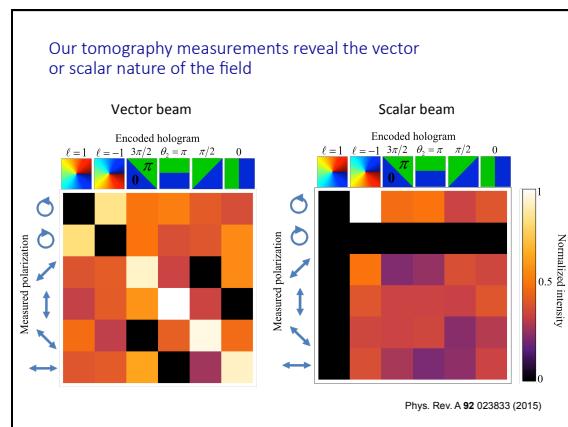
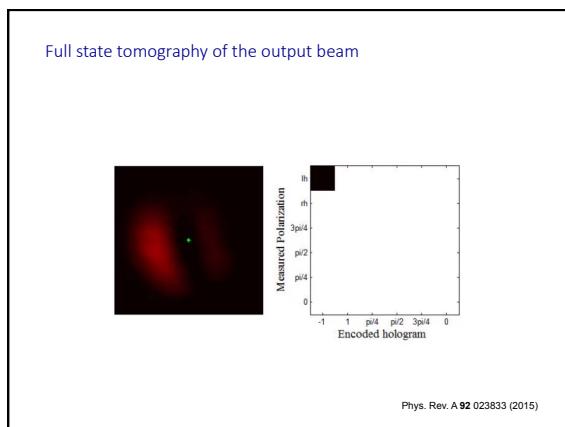
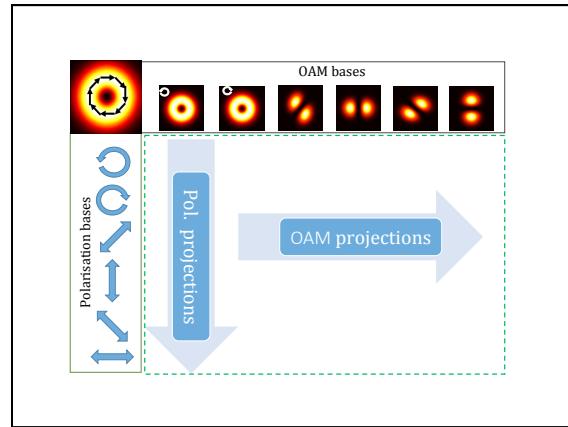
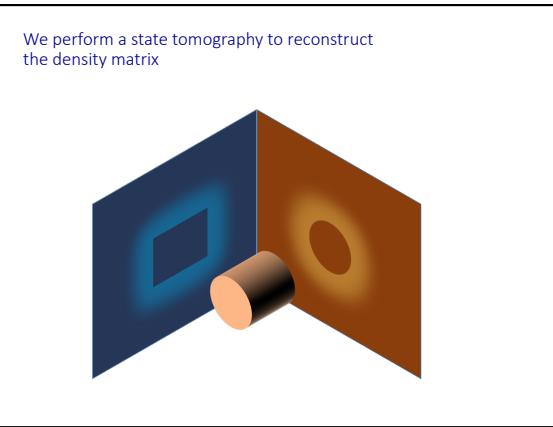
Measure 2 properties

Can we use quantum tools to describe vector beams?

Purely scalar

Purely vector

Degree of non-separability



We confirm the approach using a “tunable” vectorness

$VQF$

$\theta(^{\circ})$

$S = 2.72 \pm 0.01$

Bell inequality distinguishes classical & quantum correlations

Classical

Quantum mechanical

$-2 \leq S \leq 2$

Photon A

Photon B

Vector beams violate the Bell inequality?

Normalized on-axis intensity

Orientation of hologram,  $\theta_i$

$S = 2.72 \pm 0.01$

Classical entanglement? But can we harness it?

Walking non-separable classical light

PRL 110, 263602 (2013)

Easy to tell which from the walker distribution ... quantum is exponentially faster pace

Probability

Position ( $x$ )

Random walk

Quantum walk

