On-chip quantum frequency combs

Roberto Morandotti

Institut National de la Recherche Scientifique,
Montréal, Canada

INRS UNIVERSITÉ DE RECHERCHE
CENTRE ÉNERGIE MATÉRIAUX TÉLÉCOMMUNICATIONS
ULTRAFAST OPTICAL PROCESSING GROUP

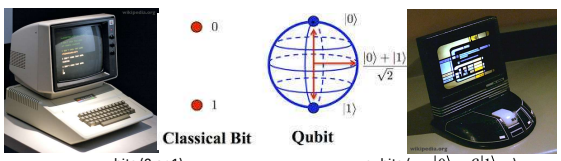
Multi-photon and high-dimensional entanglement in integrated frequency combs

Christian Reimer[1], Michael Kues[1,2], Piotr Roztocki[1], Luis Romero Cortes[1], Stefania Sciara[1,3], Benjamin Wetzel[1,4], Yanbing Zhang[1], Alfonso Cino[3], Sai T. Chu[5], Brent E. Little[6], David J. Moss[7], Lucia Caspani[1,8], José Azaña[1], and Roberto Morandotti[1,9,10]

[1]INRS-EMT, [2]Glasgow University, [3]University of Palermo, [4]Sussex University, [5]Xi'an Chinese Academy of Sciences, [6]City University of Hong Kong, [7]Swinburne University, [8]University of Strathclyde, [9]UESTC, [10]ITMO University

INRS UNIVERSITÉ DE RECHERCHE
CENTRE ÉNERGIE MATÉRIAUX TÉLÉCOMMUNICATIONS
ULTRAFAST OPTICAL PROCESSING GROUP

Classical computer vs. Quantum computer

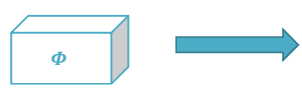


bits (0 ***or*** 1) qubits ($\alpha|0\rangle+\beta|1\rangle$)

Problems

Classical computer		Quantum computer
O{e^N}	Factorization	O{N^3}
O{N/2}	Sorting	Osqr{N}
O{N}	Linear equations	O{ln(N)}

Quantum States



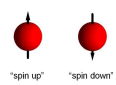
$|\Phi\rangle$

$\rho = |\Phi\rangle\langle\Phi|$

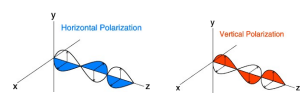
Quantum System → no classical description

Quantum State: describes all the properties of the system

Examples:

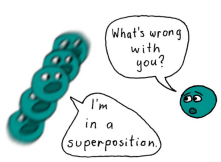


Spin (intrinsic angular momentum) → $|\uparrow\rangle, |\downarrow\rangle$



Photon polarization → $|H\rangle, |V\rangle$

Superposition of quantum states

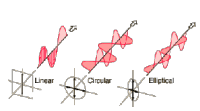


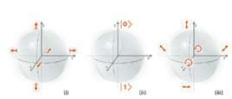
$|\circlearrowleft\rangle = \alpha|H\rangle + \beta|V\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

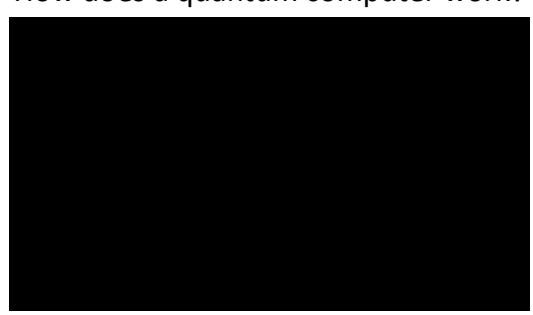
$|\nearrow\rangle = a|\uparrow\rangle + b|\downarrow\rangle$

$|a|^2 + |b|^2 = 1$





How does a quantum computer work?



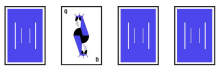
https://www.youtube.com/watch?v=g_IaVepNDT4

How does a quantum computer actually look like?

Maybe like this.....

Or maybe like this.....

Courtesy of IBM (https://www.research.ibm.com/ibm-q/quantum-card-test/)



8

Main types of quantum computation

Linear quantum computing

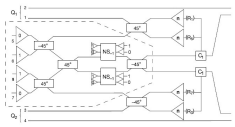
Source:
- Simple
- ➢ Indistinguishable photons

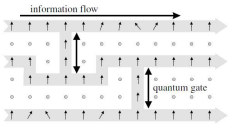
Operation:
- Complex
- ➢ One/two photon quantum gates

One-way quantum computing

Source:
- Complex
- ➢ Multi-photon entangled states

Operation:
- Simple
- ➢ Measurements





E. Knill et al. *Nature* **409**, 46-52 (2001)

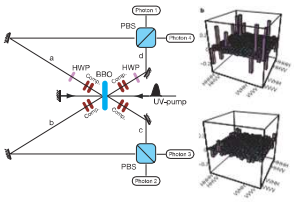
R. Raussendorf and H.J. Biegel, *PRL 86, 5188 (2001)*

9

Increase the quantum state complexity

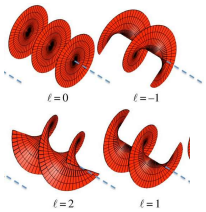
Dimensionality of Hilbert space: (Dimensionality of particles)^(Number of particles)

Multi-photon states



P. Walther et al., *Nature* **434**, 196 (2005).
R. Prevedel et al., *Nature* **445**, 65 (2007).

High-dimensional states

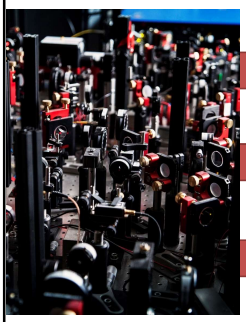


A. Mair et al., *Nature* **412**, 3123 (2001).
A.C. Dada et al., *Nature Phys.* **7**, 677 (2011)

10

Decrease the source complexity

Integrated photonics can enable compact and at the same time powerful sources

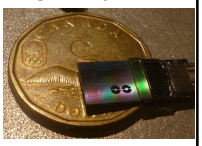


Scability

Stability

Ease-of-use

Integrated photonics

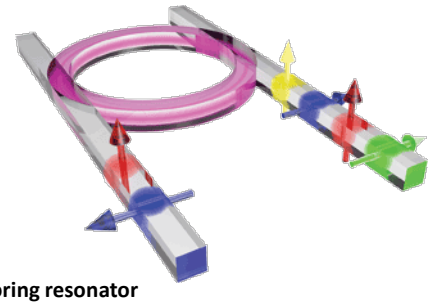


ULTRAFAST OPTICAL PROCESSING GROUP

INRS
UNIVERSITÉ DE RECHERCHE

14

Our work deals with the study and exploitation of $\chi^{(3)}$ effects in a nonlinear resonant element



Microring resonator

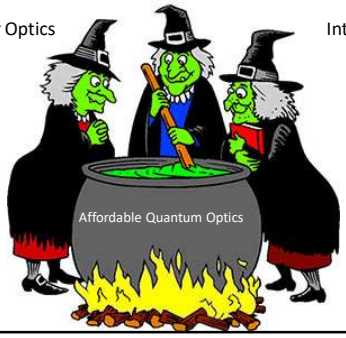
15

Our ingredients for affordable quantum optics are...

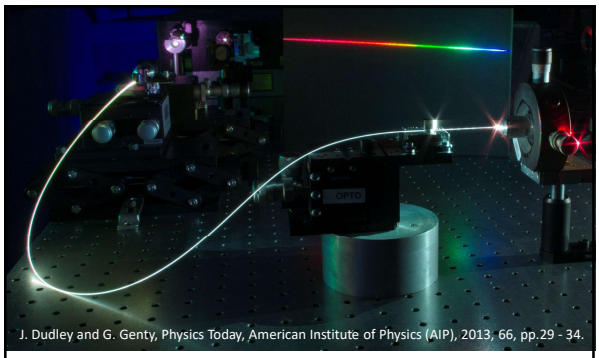
Resonant Structures

Nonlinear Optics

Integration



16



J. Dudley and G. Genty, Physics Today, American Institute of Physics (AIP), 2013, 66, pp.29 - 34.

Ingredient 1: Nonlinear Optics

Using a Taylor expansion, the polarisation density can be more generally expressed as

$$\vec{P} = \varepsilon_0 \left[\chi^{(1)} \vdots \vec{E} + \chi^{(2)} \vdots \vec{E}\vec{E} + \chi^{(3)} \vdots \vec{E}\vec{E}\vec{E} + \ldots \right]$$

$\chi^{(1)}$ = linear susceptibility (=n^2-1)

$\chi^{(2)}$ = second order susceptibility

$\chi^{(3)}$ = third order susceptibility

Laser light easily generates strong electric fields and allows nonlinearities to be observed

24

Important Nonlinear Optics Effects

Sum Frequency Generation, Difference Frequency Generation
(in this case, it is a second-order nonlinear effect)

$$\overline{E_1} = E_{10} \cdot U_1(x,y) \cdot \exp(ik_1 \cdot z - \omega_1 \cdot t)$$

$$\overline{E_2} = E_{20} \cdot U_2(x,y) \cdot \exp(ik_2 \cdot z - \omega_2 \cdot t)$$

$e^a e^b = e^{(a+b)}$
(and c.c.)

SFG: $\omega_3 = \omega_1 + \omega_2$, $k_3 = k_1 + k_2$
DFG: $\omega_3 = \omega_1 - \omega_2$, $k_3 = k_1 - k_2$

Optical Kerr effect: light-induced refractive index change
(it is a third-order nonlinear effect)

$$P_{NL} = \chi^{(3)} \cdot E^3 \sim \chi^{(3)} \mathrm{Re}[3E_0^2 E_0^* \cdot \exp(i\omega \cdot t)]$$

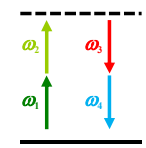
Consider only the ω terms

$$I_0 \propto |E_0|^2 = E_0 E_0^* \implies \Delta n = n_2 I_0 \propto I_0 \implies$$

Change of the imaginary part of non-linear index: two-photon absorption

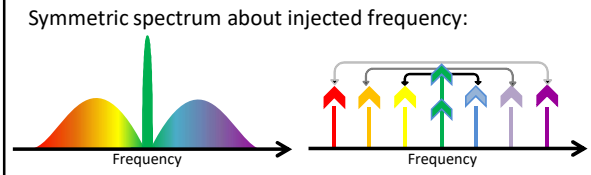
25

Four-wave-mixing arises as a result of $\chi^{(3)}$ susceptibility and involves 4 photons

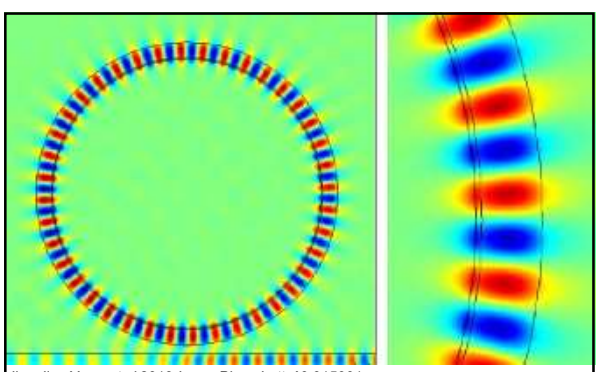


Non-degenerate case:
$\omega_1 \neq \omega_2$
Degenerate case:
$\omega_1 = \omega_2$

Symmetric spectrum about injected frequency:



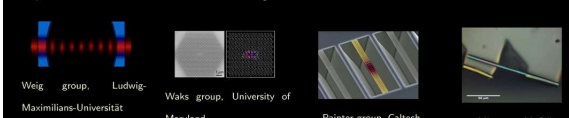
Fiber — Nonlinear resonant medium



Jing Jing Yang *et al* 2013 *Laser Phys. Lett.* **10** 015901.

Ingredient 2: Resonators

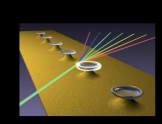
Optical resonators are very versatile!



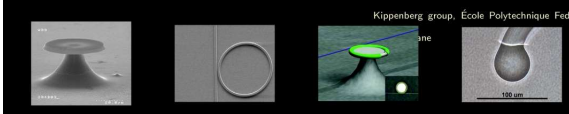
Weig group, Ludwig-Maximilians-Universität München — Waks group, University of Maryland — Painter group, Caltech — Mi group, McGill

Applications:

- Photonics devices (lasers, filters, modulators, etc.)
- Metrology (frequency combs, ultra-precise measurements)
- Sensors (refractive index, single particle, particle sizing)



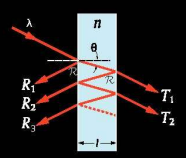
Kippenberg group, École Polytechnique Fed-ne



Verbert *et al.*, *Appl. Phys.* — Q. Xu *et al.*, *Nature* **435** 325 — L. Yang *et al.*, *Appl. Phys.* — Meldrum group, University of

62

The simplest optical resonator you can find: Fabry-Pérot



Fabry-Pérot etalon

Transmission:

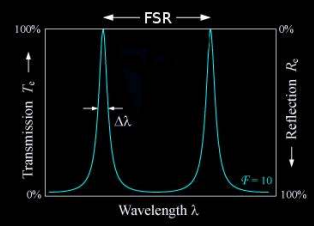
$$T = T_1 + T_2 + \ldots = \frac{1}{1 + F\sin(\delta)},$$

$$\delta = \pi\frac{\Delta\mathcal{L}}{\lambda},\ \Delta\mathcal{L} = 2nl\cos\theta,$$

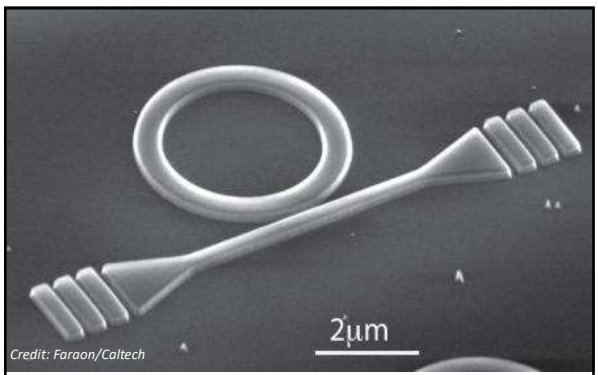
$$F = \frac{4\mathcal{R}}{(1-\mathcal{R})^2}.$$

Constructive interference at the exit: $m\lambda = \Delta\mathcal{L} \qquad (m \in \mathbb{Z})$
Each m corresponds to a different field configuration inside, a "resonant mode".

The transmission spectrum of the Fabry-Pérot resonator shows peaks.



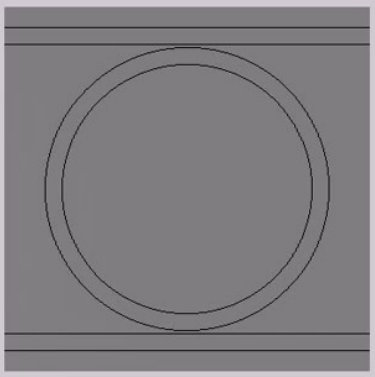
- Free Spectral Range, FSR $\approx \frac{\lambda_0^2}{\Delta\mathcal{L}}$
- Quality factor, $Q = \frac{\lambda_0}{\Delta\lambda} = \omega\tau_c$
- Cavity lifetime, τ_c
- Finesse, $\mathcal{F} = \mathrm{FSR}/(\Delta\lambda) \approx \frac{\pi\sqrt{\mathcal{R}}}{1-\mathcal{R}}$
- Intra-cavity field enhancement $\approx \frac{\mathcal{F}}{\pi\sqrt{\mathcal{R}}}$



Credit: Faraon/Caltech

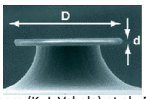
Ingredient 3: Integration

https://www.youtube.com/watch?v=m4j2y53f1sQ



$\chi^{(3)}$ materials are easier to integrate than $\chi^{(2)}$, but their low nonlinearity must be compensated with cavity enhancement

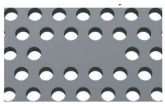
Micro-toroid



T. Kippenberg (K. J. Vahala) et al., Phys. Rev. Lett. **93**, 083904 (2004)

P. Del'Haye (R. Holzwarth, T. Kippenberg) et al., Nature **450**, 1214 (2007)

Photonic crystals

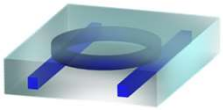


Akahane et al., Nature **425**, 944-947 (2003)

Whispering gallery mode resonators

A. Savchenkov, (A. B. Matsko) et al., Phys. Rev. Lett. **101**, 093902 (2008)

Micro-ring resonators



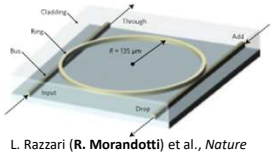
Ferrera (Morandotti) et al., Nature Photonics **2**, 737-740 (2008)

Micro-ring resonator sources have been demonstrated in a variety of integrated platforms

Silicon nitride

J. S. Levy (M.Lipson, A. Gaeta), et al., *Nature Photon*. **4**, 37 (2010)

Hydex

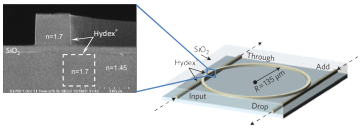


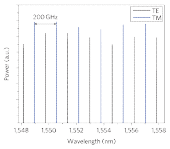
L. Razzari (**R. Morandotti**) et al., *Nature Photon*. **4**, 41 (2010)

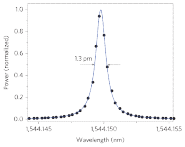
M. Ferrera (**R. Morandotti**) et al., *Nature Photon*. **2**, 737 (2008)

D.J. Moss (**R. Morandotti**) et al., *Nature Photon*. **7**, 597 (2013)

Hydex® is a CMOS-compatible high-index glass platform







- Similar to SiN, SiON
- Tightly confined mode: Δn ~ 0.25
- Cross section: 1.50x1.45 µm^2
- Linear loss < 0.06dB/cm
- High nonlinearity: γ = 220 W^{-1}km^{-1}
- FSR = 200GHz, Q = 0.3 – 1.5 million
- Resonance aligned to the ITU grid
- Linewidths compatible with quantum memories

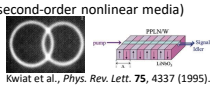
M. Ferrera (**R. Morandotti**) et al., *Nat. Photon.* **2**, 737-740 (2008)
Moss, **Morandotti**, Gaeta, Lipson, *Nat. Photon.* **7**, 597–607 (2013)

41

A variety of optics platforms have been used to demonstrate state-of-the-art quantum light sources

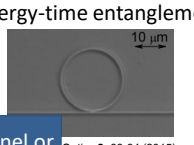
Single photon quantum sources

- Spontaneous parametric down-conversion (second-order nonlinear media)



Kwiat et al., *Phys. Rev. Lett.* **75**, 4337 (1995).

Energy-time entanglement

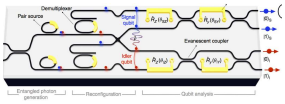


Optica **2**, 88-94 (2015)

No on-chip multi-frequency channel or multi-photon quantum sources!

- Spontaneous four-wave mixing (third-order nonlinear media)
  - Optical fibers
    Takesue et al., *Phys. Rev. A* **70**, 031802(R) (2004).
    Li et al., *Phys. Rev. Lett.* **94**, 053601 (2005).
    Dong et al., *Opt. Express* **22**, 359 (2014).
  - Integrated waveguides
    Takesue et al., *Appl Phys. Lett.* **91**, 201108 (2007).
    Olislager et al., *Opt. Lett.* **38**, 1960 (2013).

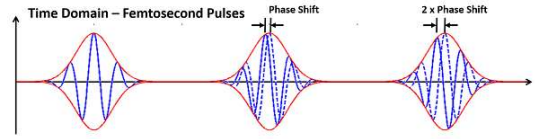
Source entanglement

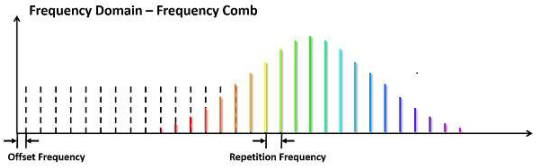


J. Silverstone et al., *Nat. Commun.* **6**, 7948 (2015)

43

Optical Frequency Combs

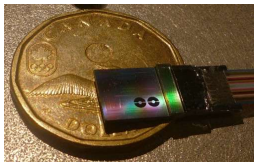




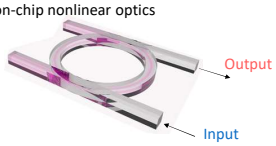
T. Udem et al., *Phys. Rev. Lett.* **82**, 3568 (1999).

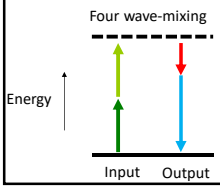
45

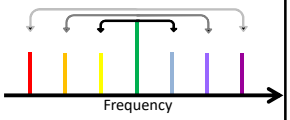
An integrated frequency comb source is created by four-wave mixing in the micro-ring



Microcavity field enhancement enables on-chip nonlinear optics

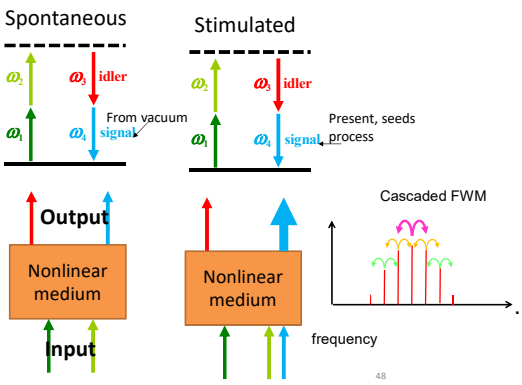






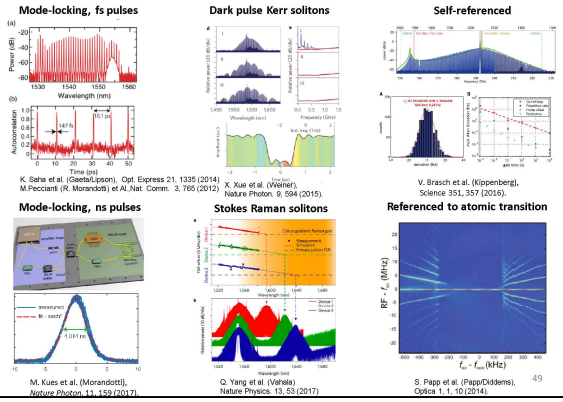
Stimulated (classical) vs spontaneous FWM (quantum)

Four-wave-mixing



48

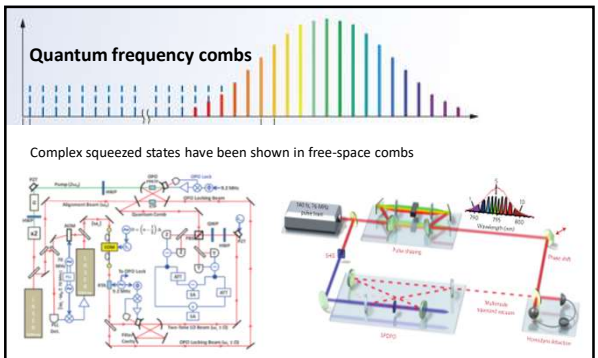
Classical on-chip optical frequency combs: a small selection



K. Saha et al. (Gaeta/Lipson), Opt. Express 21, 1335 (2014)
M.Peccianti (R. Morandotti) et Al.Nat. Comm. 3, 765 (2012)
X. Xue et al. (Weiner), Nature Photon. 9, 594 (2015).
V. Brasch et al. (Kippenberg), Science 351, 357 (2016).
M. Kues et al. (Morandotti), Nature Photon. 11, 159 (2017).
Q. Yang et al. (Vahala) Nature Physics. 13, 53 (2017)
S. Papp et al. (Papp/Diddams), Optica 1, 1, 10 (2014).

49

Quantum frequency combs

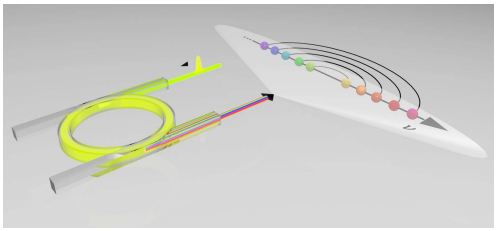


Complex squeezed states have been shown in free-space combs

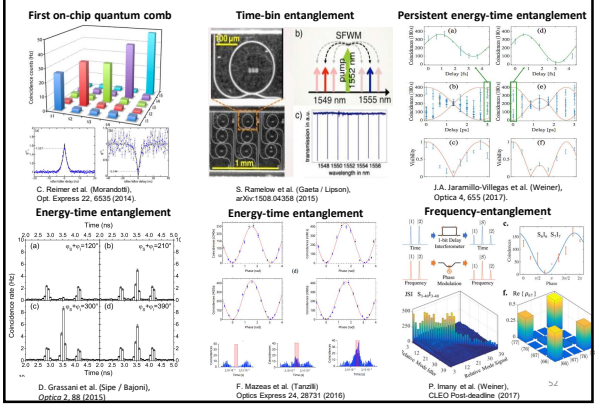
Pysher et al., *Phys. Rev. Lett.* **107**, 030505 (2011).
Chen et al., *Phys. Rev. Lett.* **112**, 120505 (2014).

J. Roslund et al., *Nature Photon.* **8**, 109 (2014).

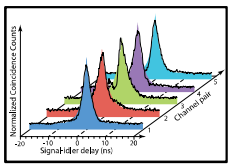
A new research direction:
Integrated quantum frequency combs



Quantum on-chip optical frequency combs: a small selection



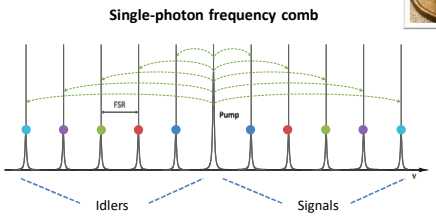
Quantum state generation via integrated frequency combs



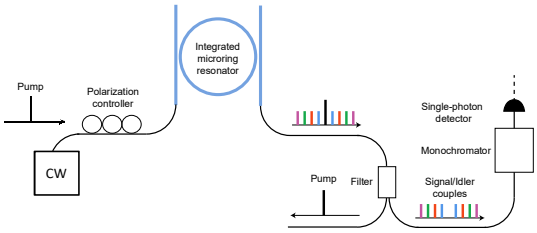
Microring resonators

Frequency comb of
- **Single photons**
- Entangled photons
- Multi-photon entangled states
- High-dimensional states

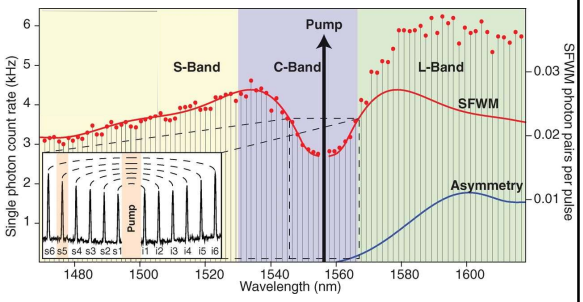
Spontaneous four-wave mixing generates photon pairs
at frequencies corresponding to the micro-ring resonances



Single-photon frequency comb: Setup (Spectrum)



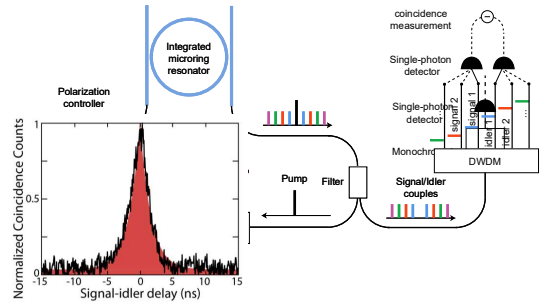
The single-photon output is broad and flat, spanning >50 channels over the S, C, L telecommunications band



C. Reimer, M. Kues, P. Roztocki (**R. Morandotti**) et al., Science **351**, 1176-1180 (2016).

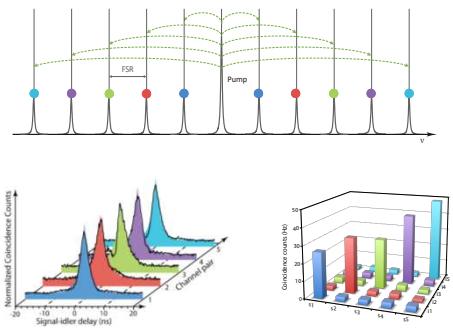
58

Single-photon frequency comb: Correlation Setup



59

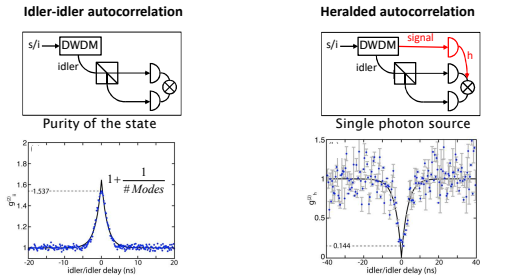
Clear coincidence peaks were observed between channel pairs centered about the pump frequency



C. Reimer (**R. Morandotti**) et al., *Opt. Express* **22**, 6535 (2014).

61

The source has close to single frequency-mode operation and a high source-purity characteristic



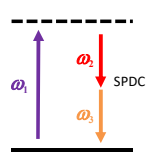
Idler-idler peak: 1.537
→ 1.86 effective modes

Heralded idler-idler dip: 0.144 ≪ 0.5
→ Good heralded source

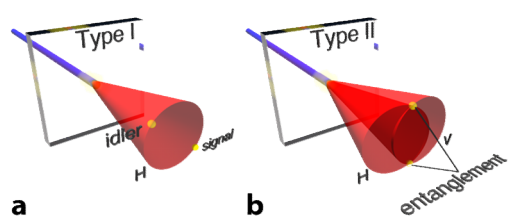
C. Reimer (**R. Morandotti**) et al., *Opt. Express* **22**, 6535 (2014).

64

Entanglement in general (for qubits)



Entanglement in (bulk) Optics



$$|\psi\rangle_{ent} = |H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2 \,,$$

Wave function and density matrix of a quantum state

Wavefunction representation

$$|\psi\rangle = \alpha|11\rangle + \beta|12\rangle + \gamma|21\rangle + \delta|22\rangle$$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

Density matrix representation

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\hat{\rho} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} \cdot (\alpha^* \ \beta^* \ \gamma^* \ \delta^*)$$

Example:

Wavefunction

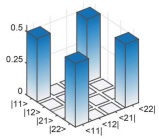
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |22\rangle)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Density matrix

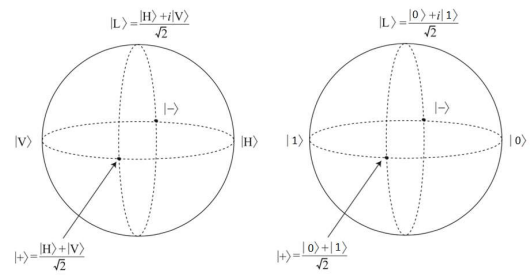
$$\hat{\rho} = \frac{1}{2} \cdot \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

Visualization



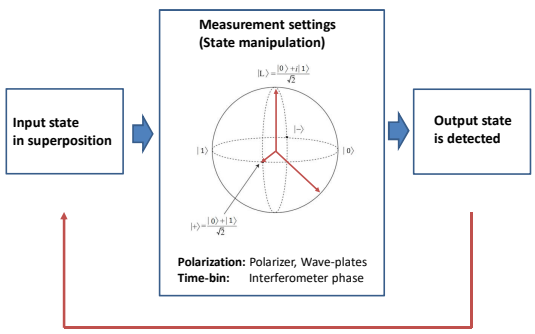
75

Visualisation of superposition: Poincaré sphere



76

Projection Measurement:



IFF a photon is detected, it was projected onto specific state, represented by a vector in the Poincaré Sphere

77

Performing a measurement (a bit of math):

A measurement can be described by a projection wave vector or an operator

Projection vector: $|\phi_P\rangle$ Projection operator: $\hat{O}_P = |\phi_P\rangle\langle\phi_P|$

The probability to measure a quantum state in this particular projection is give by:

$$P = |\langle\phi_P|\psi\rangle|^2 = \langle\psi|\phi_P\rangle\langle\phi_P|\psi\rangle$$
$$= \langle\psi|\hat{O}_P|\psi\rangle = \langle\hat{O}_P\rangle$$

An operator can be used to describe several measurements in a compact form:

$$\hat{O'}_P = \sum_n \lambda_n \cdot |\phi_n\rangle\langle\phi_n|$$

The expectation value $\langle\hat{O'}_P\rangle$ is given by the sum of the individual (scaled) probabilities

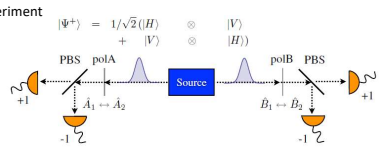
$$\langle\hat{O'}_P\rangle = \langle\psi|(\textstyle\sum_n \lambda_n \cdot |\phi_n\rangle\langle\phi_n|)|\psi\rangle = \sum_n \lambda_n \cdot P_n$$

Important: The expectation value of an operator can be negative or larger than one

78

Bell Inequality

Bell-type experiment



M. C Tichy et al., *J. Phys. B: At. Mol. Opt. Phys.* **44**, 192001 (2011)

$\hat{A}_1, \hat{A}_2$: Alice's measurement settings

$\hat{B}_1, \hat{B}_2$: Bob's measurement settings

$$\boxed{\langle\hat{S}\rangle = |\langle\hat{A}_1 \otimes \hat{B}_1\rangle + \langle\hat{A}_1 \otimes \hat{B}_2\rangle + \langle\hat{A}_2 \otimes \hat{B}_1\rangle - \langle\hat{A}_2 \otimes \hat{B}_2\rangle| \leq 2}$$

Local Realism: There are NO measurement settings that can violate the inequality

Nonlocality: There ARE measurement settings that can violate the inequality

79

Can we trick Bell inequalities (if we assume local realism)?

$$\boxed{\langle S\rangle = |\langle\hat{A}_1 \otimes \hat{B}_1\rangle + \langle\hat{A}_1 \otimes \hat{B}_2\rangle + \langle\hat{A}_2 \otimes \hat{B}_1\rangle - \langle\hat{A}_2 \otimes \hat{B}_2\rangle| \leq 2}$$

$-\langle\hat{A}_2 \otimes \hat{B}_2\rangle = +1 \rightarrow$ $\hat{A}_2$=-1 and $\hat{B}_2$=+1

$\langle\hat{A}_2 \otimes \hat{B}_1\rangle = +1 \rightarrow \hat{B}_1$=-1

$\langle\hat{A}_1 \otimes \hat{B}_1\rangle = +1 \rightarrow \hat{A}_1$= -1

But then.....

$\langle\hat{A}_1 \otimes \hat{B}_2\rangle = (-1) \cdot (+1) = -1$

80

A bit more on Bell inequalities

Bell Operator: $\hat{S} = \hat{A}_1 \otimes \hat{B}_1 + \hat{A}_1 \otimes \hat{B}_2 + \hat{A}_2 \otimes \hat{B}_1 - \hat{A}_2 \otimes \hat{B}_2$

Measurement outcome is given by the expectation value of the operator: $\langle \hat{S} \rangle$

Trick: The expectation value of an operator is equal or smaller than the root of the expectation value of the its square operator

$$\langle \hat{S} \rangle \leq \sqrt{\langle \hat{S}^2 \rangle}$$

$$\langle \hat{S}^2 \rangle = 4 - [\hat{A}_1, \hat{A}_2] \otimes [\hat{B}_1, \hat{B}_2] \text{ with } \hat{A}_1 \cdot \hat{A}_1 = 1,\ \hat{B}_1 \cdot \hat{B}_1 = 1, \text{ etc.}$$

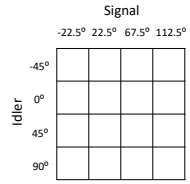
Local Realism	Quantum nonlocality
$[\hat{A}_1, \hat{A}_2] = 0$	$[\hat{A}_1, \hat{A}_2] = -2 \ldots 0 \ldots +2$
$[\hat{B}_1, \hat{B}_2] = 0$	$[\hat{B}_1, \hat{B}_2] = -2 \ldots 0 \ldots +2$
$\langle \hat{S} \rangle \leq 2$	$\langle \hat{S} \rangle \leq 2\sqrt{2}$

L. J. Lindau, Phys. Lett. A **120**, 54 (1987).

81

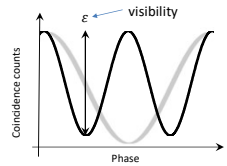
Measuring Bell inequality violations

Direct measurement



- Min. 16 measurements required
- Optimal measurement settings have to be used
- $\langle \hat{S} \rangle$ value can be directly calculated
- $\langle \hat{S} \rangle > 2$ violates inequality

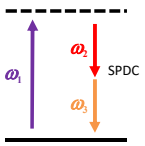
Measurement with quantum interference



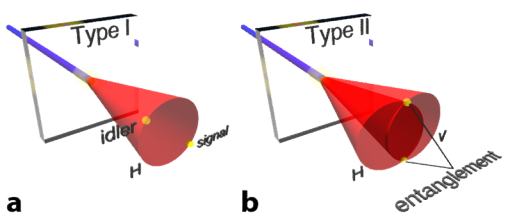
- Both polarizers are rotated together
- Period doubles compared to classical interference
- $\langle \hat{S} \rangle = 2\sqrt{2} * \varepsilon$ value can be calculated indirectly making use of the Linear Noise Model.
- Visibility $\varepsilon > \frac{1}{\sqrt{2}} \approx 71\%$ violates Bell inequality

J.F. Clauser et al. (CHSH), Phys Rev. Lett. **24**, 549 (1970).
D. Colins et al., Phys. Rev. Lett **88**, 040404 (2002)

82

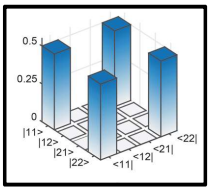


Entanglement in (bulk) Optics



$$|\psi\rangle_{ent} = |H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2 \,,$$

Quantum state generation via integrated frequency combs

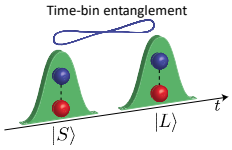


Microring resonators

Frequency comb of
- Single photons
- **Entangled photons**
- Multi-photon entangled states
- High-dimensional states

Complex quantum states

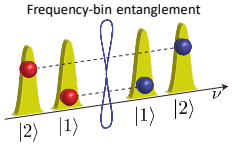
Time-bin entanglement



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|SS\rangle + |LL\rangle)$$

Brendel et al., *Phys. Rev. Lett.* **82**, 2594 (1999).
Marcikic et al., *Phys Rev. A* **66**, 062308 (2002).

Frequency-bin entanglement



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |22\rangle)$$

Olislager, L. et al., *Phys. Rev. A* **82,** 013804 (2010).
Bernhard, C. et al., *Phys. Rev. A* **88,** 032322 (2013).

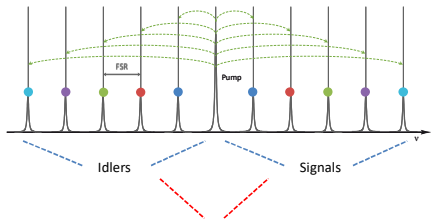
On-chip generation of

Multi-photon entangled states

High-dimensional entangled states

85

To generate time-bin entanglement, we considered a pair at a time



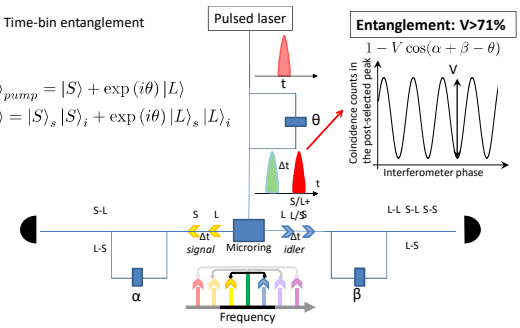
For example, those two (or any other)

86

Generation of time-bin entangled photon pairs

Time-bin entanglement

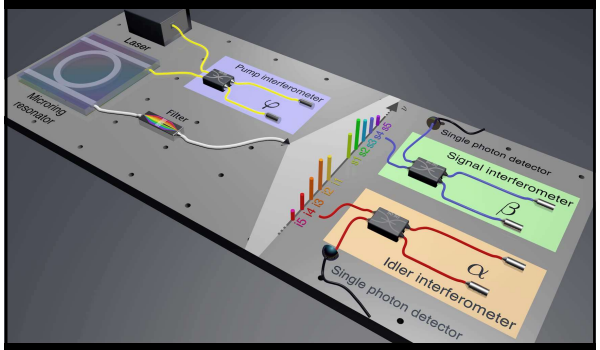
$$|\psi\rangle_{pump} = |S\rangle + \exp(i\theta)\,|L\rangle$$
$$|\psi\rangle = |S\rangle_s\,|S\rangle_i + \exp(i\theta)\,|L\rangle_s\,|L\rangle_i$$



Brendel et al., *Phys. Rev. Lett.* **82**, 2594 (1999).
Marcikic et al., *Phys Rev. A* **66**, 062308 (2002).

87

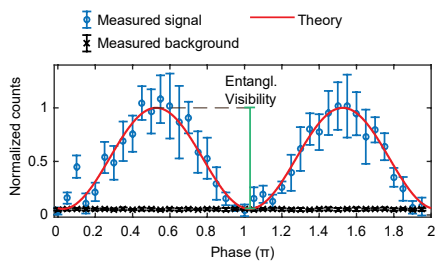
Generation of time-bin entangled photon pairs



Video made by Michael Kues (Morandotti group)

88

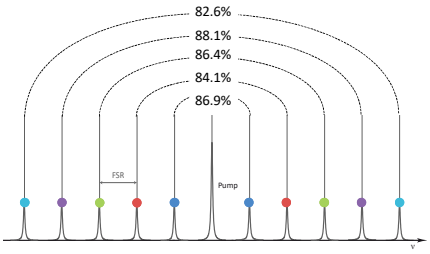
Quantum interference visibility of 89.3% (99.1% background subtracted) verifies entangled photon pairs



C. Reimer, M. Kues, P. Roztocki (**R. Morandotti**) et al., Science **351**, 1176-1180 (2016).

90

Time-bin entanglement was demonstrated on 5 resonance pairs tested, likely spans more telecommunications bands



Frequency comb of entangled photon pairs

C. Reimer, M. Kues, P. Roztocki (**R. Morandotti**) et al., Science **351**, 1176-1180 (2016).

91

Measurement of density matrix: tomography

Experimental reconstruction of the density matrix can be achieved by means of multiple projection measurements

First-photon setting | **Second-photon setting**

$|L\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ → $|L\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ → $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

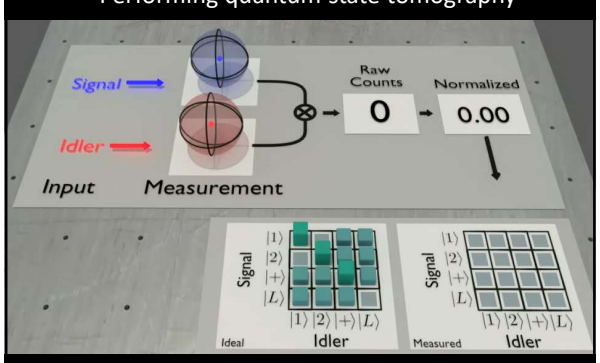
$|0\rangle$ → $|0\rangle$

$|1\rangle$ → $|1\rangle$

D. F.V. James et al., Phys Rev. A **64**, 052312 (2001).

92

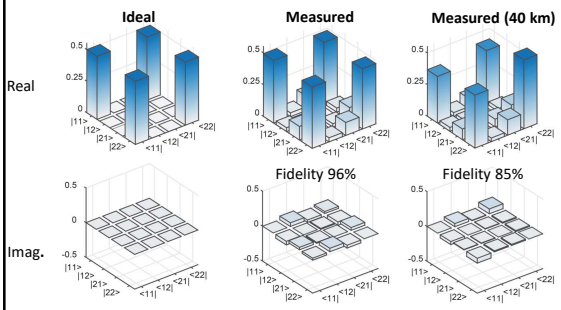
Performing quantum state tomography



Video made by Benjamin Maclellan (Morandotti group)

93

Quantum state tomography confirms a high fidelity (0.96), high purity (1.0) entangled qubit source

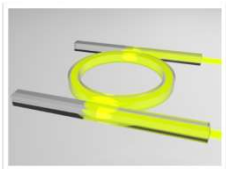


$$F = \left(\mathrm{Tr}\left(\left[\sqrt{\rho_{th}}\,\rho_{exp}\sqrt{\rho_{th}}\right]^{1/2}\right)\right)^2$$

C. Reimer, M. Kues, P. Roztocki (**R. Morandotti**) et al., Science **351**, 1176-1180 (2016).

94

Quantum state generation via integrated frequency combs



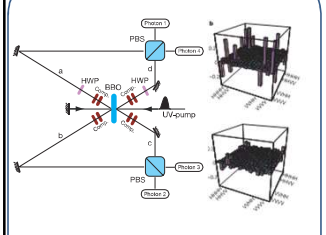
Microring resonators

Frequency comb of
- Single photons
- Entangled photons
- **Multi-photon entangled states**
- High-dimensional states

Increase the quantum state complexity

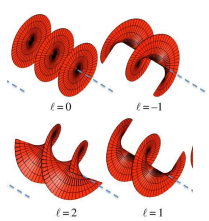
Dimensionality of Hilbert space: (Dimensionality of particles)^(Number of particles)

Multi-photon states



P. Walther et al., *Nature* **434**, 196 (2005).
R. Prevedel et al., *Nature* **445**, 65 (2007).

High-dimensional states

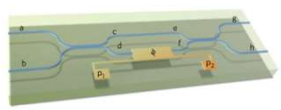


A. Mair et al., *Nature* **412**, 3123 (2001).
A.C. Dada et al., *Nature Phys.* **7**, 677 (2011)

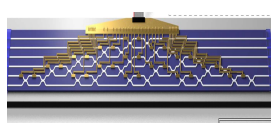
97

Multi-photon entanglement on a chip

Quantum state control

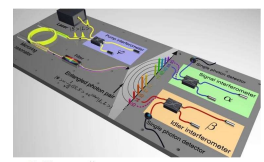


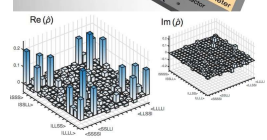
J.C.F. Matthews et al., *Nature Photon* **3**, 346 (2009).



J. Carolan et al., *Science* **349**, 711 (2016).

Quantum state generation





C. Reimer (**Morandotti**) et al., *Science* **351**, 1175 (2016).

98

Product of Bell States

Most multi-photon entangled states can be generated from a product of Bell states. Their generation is therefore a critical first step to achieve a multitude of states.

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle) \qquad |\Phi_2\rangle = \frac{1}{\sqrt{2}}(|0_C 0_D\rangle + |1_C 1_D\rangle)$$

$$|\Psi\rangle = |\Phi_1\rangle \otimes |\Phi_1\rangle = \frac{1}{2}(|0_A 0_B 0_C 0_D\rangle + |0_A 0_B 1_C 1_D\rangle + |1_A 1_B 0_C 0_D\rangle - |1_A 1_B 1_C 1_D\rangle)$$

Phase Rotation e.g. Nature 429, 159 (2004)

Selective Beam Splitter e.g. Nature 403, 515 (2000)

Two important examples:

GHZ state (e.g. for quantum metrology):

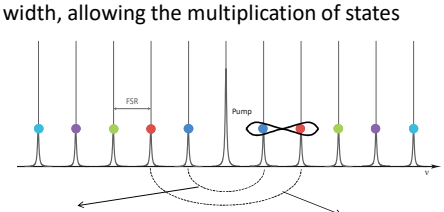
$$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B 0_C 0_D\rangle + |1_A 1_B 1_C 1_D\rangle)$$

Cluster state (e.g. for one-way quantum computing):

$$|C_4\rangle = \frac{1}{2}(|0_A 0_B 0_C 0_D\rangle + |0_A 0_B 1_C 1_D\rangle + |1_A 1_B 0_C 0_D\rangle - |1_A 1_B 1_C 1_D\rangle)$$

99

Due to a **similar resonance bandwidth** over the comb, time-bin widths are **equal** and matched to the excitation field width, allowing the multiplication of states



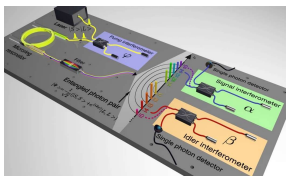
$$|\psi_1\rangle = |S\rangle_{s1}|S\rangle_{i1} + \exp(i\theta)|L\rangle_{s1}|L\rangle_{i1} \qquad |\psi_2\rangle = |S\rangle_{s2}|S\rangle_{i2} + \exp(i\theta)|L\rangle_{s2}|L\rangle_{i2}$$

$$|\psi_{4p}\rangle = |\psi_1\rangle \bigotimes |\psi_2\rangle$$

$$|\psi_{4p}\rangle = |S\rangle_{s1}|S\rangle_{i1}|S\rangle_{s2}|S\rangle_{i2} + |L\rangle_{s1}|L\rangle_{i1}|S\rangle_{s2}|S\rangle_{i2} + |S\rangle_{s1}|S\rangle_{i1}|L\rangle_{s2}|L\rangle_{i2} + |L\rangle_{s1}|L\rangle_{i1}|L\rangle_{s2}|L\rangle_{i2}$$

100

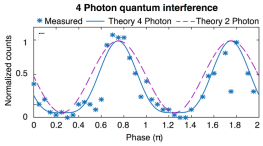
On-chip generation of 4-photon quantum state



Photon measurement

4 photon detection rate: 0.17 Hz

4 photon generation rate: 135kHz (considering 14.75 dB loss)

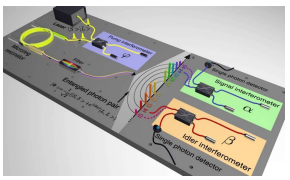


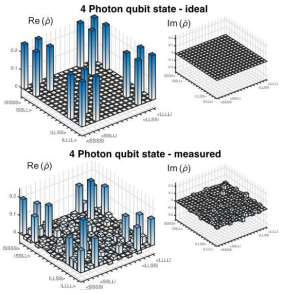
→ Four photon interference proves entangled quantum state generation

C. Reimer, M. Kues, P. Roztocki (**R. Morandotti**) et al., Science **351**, 1176-1180 (2016).

102

On-chip generation of 4-photon quantum state





Tomographic reconstruction of state density matrix

$$|\psi_{4p}\rangle = |SSSS\rangle + |SSLL\rangle + |LLSS\rangle + |LLLL\rangle$$

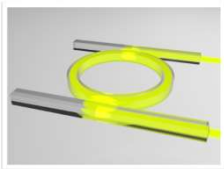
Fidelity: 64%

→ First generation of multi-photon states on-chip

C. Reimer, M. Kues, P. Roztocki (**R. Morandotti**) et al., Science **351**, 1176-1180 (2016).

103

Quantum state generation via integrated frequency combs



Microring resonators

Frequency comb of

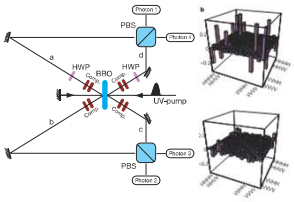
- Single photons
- Entangled photons
- Multi-photon entangled states
- **High-dimensional states**

Entanglement of Qudits

Increase the quantum state complexity

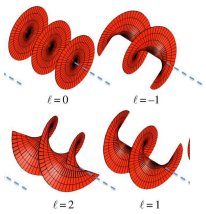
Dimensionality of Hilbert space: (Dimensionality of particles)^(Number of particles)

Multi-photon states



P. Walther et al., *Nature* **434**, 196 (2005).
R. Prevedel et al., *Nature* **445**, 65 (2007).

High-dimensional states

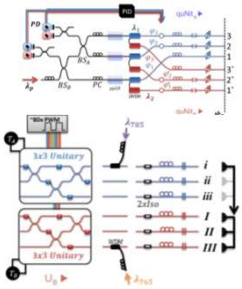


A. Mair et al., *Nature* **412**, 3123 (2001).
A.C. Dada et al., *Nature Phys.* **7**, 677 (2011)

106

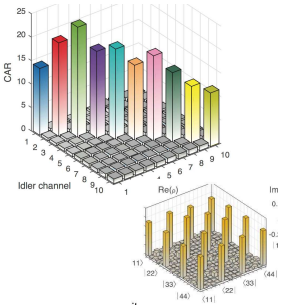
High-dimensional entanglement on a chip

Quantum state control



C. Schaeff et al., *Optica* **2**, 523 (2015).

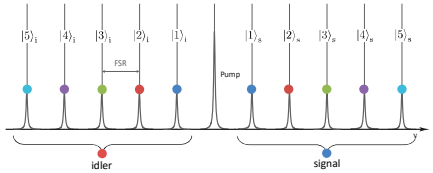
Quantum state generation



M. Kues (**Morandotti**) et al. *Nature* **546**, 622-626 (2017).

107

On-chip high-dimensional quantum states - concept



Two-photon high-dimensional quDit state

$$|\psi\rangle = \frac{1}{\sqrt{D}}(|1\rangle_s|1\rangle_i + |2\rangle_s|2\rangle_i + |3\rangle_s|3\rangle_i + \ldots)$$

$$|\psi\rangle = \frac{1}{\sqrt{D}}\sum_{k=1}^{D}|k\rangle_s|k\rangle_i$$

→ Frequency-entangled state

M. Kues, C. Reimer, P. Roztocki (**R. Morandotti**) et al., Nature **546**, 622-626 (2017).

108

Schmidt decomposition

There are many ways to describe the same quantum state, but each way has to use a minimum number of modes to do so. This number is called the Schmidt number.

$$\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) = \frac{1}{\sqrt{2}}(|\nearrow\nearrow\rangle + |\swarrow\swarrow\rangle)$$

$$|\nearrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \qquad |\swarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$$

$$\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) = \frac{1}{\sqrt{2}}(|\circlearrowleft\circlearrowleft\rangle + |\circlearrowright\circlearrowright\rangle)$$

$$|\circlearrowleft\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle) \qquad |\circlearrowright\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle)$$

Schmidt decomposition → Schmidt coefficient: $\frac{1}{\sqrt{2}}$
Schmidt number: 2

Higher dimension (d=3)

$$\frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$$ → Schmidt coefficient: $\frac{1}{\sqrt{3}}$
Schmidt number: 3

Schmidt number *K*: number of non-vanishing Schmidt coefficients

109

Schmidt decomposition: more examples

D=2

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$ → Schmidt number: 2
Maximally entangled state

$$\frac{1}{5}(3|00\rangle + 4|11\rangle)$$ → Schmidt number: 2
Entangled state (not maximum)

D=3

$$\frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$$ → Schmidt number: 3
Maximally entangled state

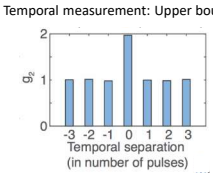
$$\frac{1}{\sqrt{94}}(2|00\rangle + 8|11\rangle + 5|22\rangle)$$ → Schmidt number: 3
Entangled state (not maximum)

The Schmidt number witnesses the dimensionality of the state but NOT the quantity of entanglement.

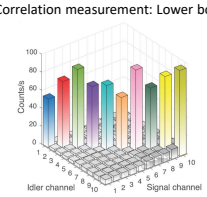
110

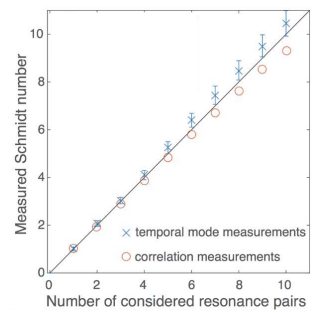
Lower and upper bound for the Schmidt number

Temporal measurement: Upper bound



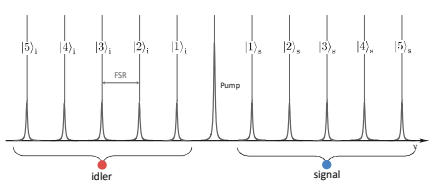
Correlation measurement: Lower bound





113

Frequency-bin quantum states:



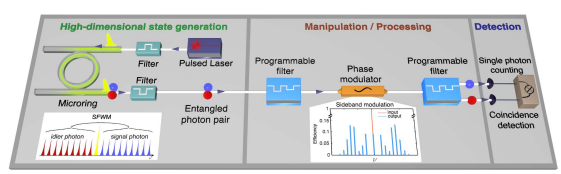
High-dimensional quDit state: 5x5 = 25 dimensions for the state above,
Min 10 x 10 = 100 dimensions in our system!

However:
The Schmidt mode decomposition determined only the dimensionality of the state, but not the quality of entanglement.

To measure entanglement and to perform quantum information processing, **coherent high-dimensional state manipulation is required**.

114

Quantum coherence measurement: Setup



Merging the fields of quantum state manipulation and ultrafast optical signal processing

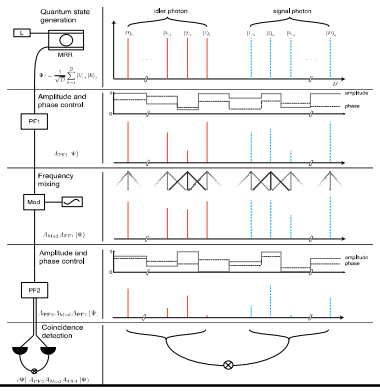
- optical phase gates for manipulating quDits → programmable phase filters
- coherent mixing of multiple modes → frequency conversion in electro-optic modulators

Using this manipulation scheme allowed to design well-defined quantum operations, for e.g. Bell-test measurements and quantum state tomography

M. Kues, C. Reimer, P. Roztocki (**R. Morandotti**) et al., Nature **546**, 622-626 (2017).

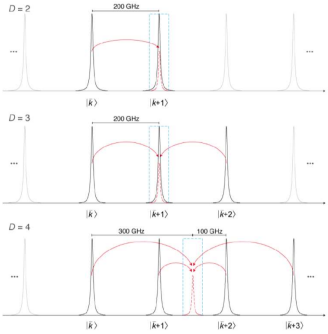
115

Generation and coherent control concept



116

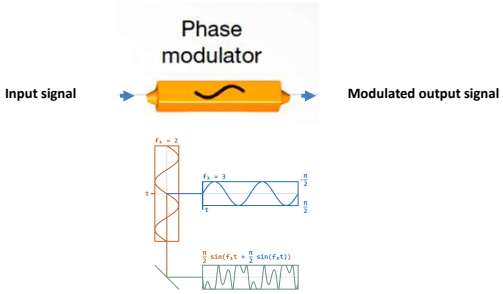
Deterministic frequency mixing with EO modulation



This term is fundamental in loosing the information on the "origin" of the frequency bin: Classical information 'kills' quantum interference!

117

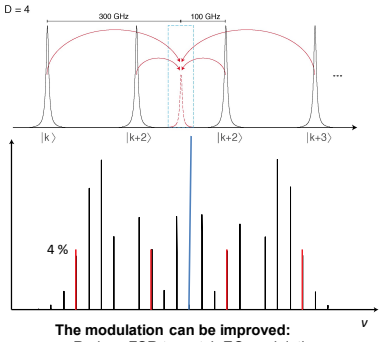
Deterministic frequency mixing with EO modulation



New frequencies are generated!

118

Deterministic frequency mixing with EO modulation

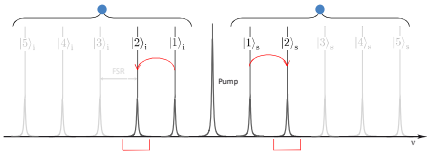


The modulation can be improved:

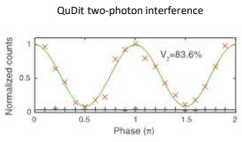
- Reduce FSR to match EO modulation
- Use tailored RF waveforms

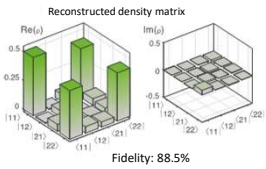
119

Frequency-entangled two-photon quDit state D=2



$$|\psi_{\text{proj.}}\rangle = |1\rangle + \alpha_2 \exp(i\varphi_2)\,|2\rangle$$





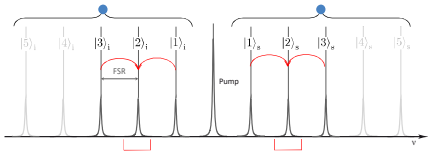
V_2=83.6%>71% → violation of Bell-inequality

Fidelity: 88.5%

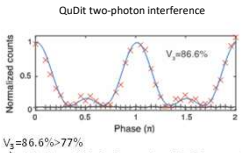
M. Kues, C. Reimer, P. Roztocki (**R. Morandotti**) et al., Nature **546**, 622-626 (2017).

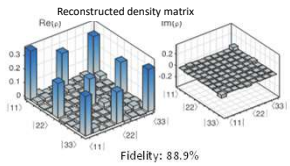
121

Frequency-entangled two-photon quDit state D=3



$$|\psi_{\text{proj.}}\rangle = |1\rangle + \alpha_2 \exp(i\varphi_2)\,|2\rangle + \alpha_3 \exp(i\varphi_3)\,|3\rangle$$





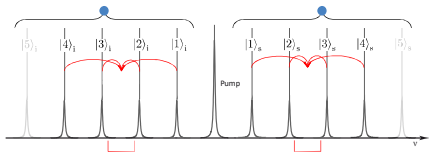
V_3=86.6%>77%
→ violation of high-dimensional Bell-inequality

Fidelity: 88.9%

M. Kues, C. Reimer, P. Roztocki (**R. Morandotti**) et al., Nature **546**, 622-626 (2017).

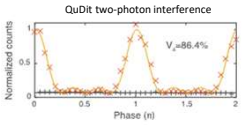
122

Frequency-entangled two-photon quDit state D=4

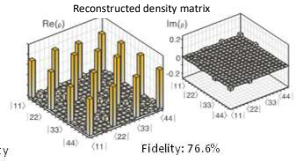


$$|\psi_{\mu\nu j}\rangle = |1\rangle + \alpha_2 \exp(i\varphi_2)|2\rangle + \alpha_3 \exp(i\varphi_3)|3\rangle + \alpha_4 \exp(i\varphi_4)|4\rangle$$

QuDit two-photon interference



Reconstructed density matrix



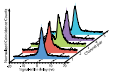
V_d=86.4%>81.7%
→ violation of high-dimensional Bell-inequality

Fidelity: 76.6%

M. Kues, C. Reimer, P. Roztocki (**R. Morandotti**) et al., Nature **546**, 622-626 (2017).

123

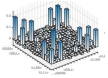
Quantum state generation via integrated frequency combs



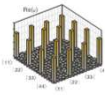
Frequency comb of:

› Single photons

› Entangled photons

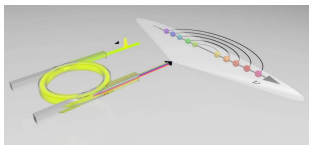


› Multi-photon entangled states



› High-dimensional entangled states

Quantum frequency combs: Foundation to generate multiple, large, and controllable quantum states on a chip



Quantum Combs:

M. Kues, C. Reimer, P. Roztocki et al., ***Nature*** 546, 622-626 (2017)
P. Roztocki, M. Kues, C. Reimer et al., ***Opt. Express*** 25, 18940 (2017)
C. Reimer, M. Kues, P. Roztocki et al., ***Science*** 351, 1176-1180 (2016)
L. Caspani, C. Reimer, M. Kues et al., ***Nanophotonics*** (2016)
C. Reimer, M. Kues, L. Caspani et al., ***Nature Commun.*** 6, 8236 (2015)
C. Reimer, L. Caspani, M. Clerici et al., ***Opt. Express*** 22, 6535 (2014)

Classical Combs:

M. Kues, C. Reimer et al, ***Nature Photon.*** 11, 159 (2017)
D.J. Moss, R. Morandotti et al., ***Nature Photon.*** 7, 597 (2013)
L. Razzari, D. Duchesne et al., ***Nature Photon.*** 4, 41 (2010)
M. Ferrera, L. Razzari et al., ***Nature Photon.*** 2, 737 (2008)

125

Thanks to our sponsors!

NSERC CRSNG
Fonds de recherche sur la nature et les technologies Québec
Économie, Innovation et Exportations Québec
Chaires de recherche du Canada / Canada Research Chairs
Canada
Bourses d'études supérieures du Canada Vanier / Canada Graduate Scholarships
PRIMA
MARIE CURIE ACTIONS

126

The *END*

(And they measured happily ever after)

129